Solutions: 7.1 (odd-numbered problems)

1. $\sin t \cot t = \sin t \cdot \frac{\cos t}{\sin t} = \cos t$

5.
$$\tan^2 x - \sec^2 x = \frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{\sin^2 x - 1}{\cos^2 x} = \frac{-\cos^2 x}{\cos^2 x} = -1$$

11.
$$\frac{1+\cos y}{1+\sec y} = \frac{1+\cos y}{1+\frac{1}{\cos y}} = \frac{(\cos y)(1+\cos y)}{\cos y + 1} = \cos y$$

15.
$$\frac{1+\csc x}{\cos x + \cot x} = \frac{1+\frac{1}{\sin x}}{\cos x + \frac{\cos x}{\sin x}} = \frac{1+\frac{1}{\sin x}}{(\cos x)\left(1+\frac{1}{\sin x}\right)} = \frac{1}{\cos x} = \sec x$$

43.
$$\begin{aligned}
(\cot x - \csc x)(\cos x + 1) &= \cot x \cos x + \cot x - \csc x \cos x - \csc x \\
&= \frac{\cos x}{\sin x}\cos x + \frac{\cos x}{\sin x} - \frac{1}{\sin x}\cos x - \frac{1}{\sin x} \\
&= \frac{\cos^2 x - 1}{\sin x} \\
&= \frac{-\sin^2 x}{\sin x} \\
&= -\sin x
\end{aligned}$$

49. If this *were* to be true, then we could cross-multiply and get
$$\sin^2\alpha = (1+\cos\alpha)(1-\cos\alpha) = 1 - \cos^2\alpha$$
or
$$\cos^2\alpha + \sin^2\alpha = 1$$
which we know is true. So, to show that our initial inequality is true, we simply work backwards, from something that is true:
$$\begin{aligned}
\sin^2\alpha + \cos^2\alpha &= 1 \\
\sin^2\alpha &= 1 - \cos^2\alpha \\
\sin^2\alpha &= (1+\cos\alpha)(1-\cos\alpha) \\
\sin\alpha &= \frac{(1+\cos\alpha)(1-\cos\alpha)}{\sin\alpha} \\
\frac{\sin\alpha}{1+\cos\alpha} &= \frac{1-\cos\alpha}{\sin\alpha}
\end{aligned}$$

63. Again, if this *were* to be true, then we could cross-multiply and get
$$\csc x - \cot x = \cot x(\sec x - 1) = \cot x \sec x - \cot x = \csc x - \cot x$$
which is true (it's equality!). So to get what we want, we simply go backwards:
$$\begin{aligned}
\csc x - \cot x &= \csc x - \cot x \\
\frac{\cos x}{\cos x}\csc x - \cot x &= \csc x - \cot x \\
\sec x \cot x - \cot x &= \csc x - \cot x \\
\cot x(\sec x - 1) &= \csc x - \cot x \\
\cot x &= \frac{\csc x - \cot x}{\sec x - 1}
\end{aligned}$$

85. When we substitute we get

$$\begin{aligned}\frac{\sin\theta}{\sqrt{1-\sin^2\theta}} &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta\end{aligned}$$

91. Graphy. This is indeed an identity, as

$$\cos^2 x - \sin^2 x = (1-\sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$$