Solutions: 7.2 (odd-numbered problems)

11. Since $\frac{1}{2}\sin 2x = \sin x \cos x$, then we have

$$\begin{aligned}
\cos^4 x \sin^4 x &= \frac{1}{16}\sin^4 2x \\
&= \frac{1}{16}\left(\frac{1-\cos 4x}{2}\right)^2 \\
&= \frac{1}{16}\left(\frac{1-2\cos 4x+\cos^2 4x}{4}\right) \\
&= \frac{1}{16}\left(\frac{1-2\cos 4x+\left(\frac{1+\cos 8x}{2}\right)}{4}\right) \\
&= \frac{1}{16}\left(\frac{3-4\cos 4x+\cos 8x}{8}\right) \\
&= \frac{1}{128}(3-4\cos 4x+\cos 8x)
\end{aligned}$$

23. $2\sin 18 \cos 18 = \sin 36$.

33. Well, $\sec x = \frac{3}{2}$, so $\cos x = \frac{2}{3}$ and

$$\sin x = -\sqrt{1-\cos^2 x} = -\sqrt{1-\left(\frac{2}{3}\right)^2} = -\sqrt{1-\frac{4}{9}} = -\frac{\sqrt{5}}{3}$$

so we have (notice the signs!)

$$\begin{aligned}
\sin\frac{x}{2} &= +\sqrt{\frac{1-\cos x}{2}} = +\sqrt{\frac{\frac{1}{3}}{2}} = +\sqrt{\frac{1}{6}} \\
\cos\frac{x}{2} &= -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{\frac{5}{3}}{2}} = -\sqrt{\frac{5}{6}} \\
\tan\frac{x}{2} &= \frac{1-\cos x}{\sin x} = \frac{1-\frac{2}{3}}{-\frac{\sqrt{5}}{3}} = \\
&= -\frac{1}{\sqrt{5}}
\end{aligned}$$

55. We have

$$\begin{aligned}
(\sin x + \cos x)^2 &= \sin^2 x + 2\sin x \cos x + \cos^2 x \\
&= (\sin^2 x + \cos^2 x) + \sin 2x \\
&= 1 + \sin 2x
\end{aligned}$$