

Solutions: 7.2 (odd-numbered problems)

Recall that \mathbb{Z} means the integers . . .

5.

$$\begin{aligned}2 \sin x + \sqrt{3} &= 0 \\ \rightarrow \sin x &= -\frac{\sqrt{3}}{2} \\ \rightarrow x &= \left\{ \frac{4\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \right\}\end{aligned}$$

15.

$$\begin{aligned}\cos x \sin x - 2 \cos x &= 0 \\ \rightarrow \cos x(\sin x - 2) &= 0\end{aligned}$$

This gives us two equations:

$$\begin{aligned}\cos x &= 0 \\ \sin x - 2 &= 0\end{aligned}$$

The second equation has no solution, but the solution to the equation $\cos x = 0$ is

$$\left\{ \frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \right\}$$

35.

$$\begin{aligned}4 \sin x \cos x + 2 \sin x - 2 \cos x - 1 &= 0 \\ \rightarrow 2 \sin x(2 \cos x + 1) - (2 \cos x + 1) &= 0 \\ \rightarrow (2 \sin x - 1)(2 \cos x + 1) &= 0\end{aligned}$$

This gives us two equations:

$$\left. \begin{aligned}2 \sin x - 1 &= 0 \\ \sin x &= \frac{1}{2} \\ x &= \left\{ \frac{\pi}{6} + 2k\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{6} + 2k\pi \mid k \in \mathbb{Z} \right\}\end{aligned} \right| \left. \begin{aligned}2 \cos x + 1 &= 0 \\ \cos x &= -\frac{1}{2} \\ x &= \left\{ \frac{2\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{4\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \right\}\end{aligned} \right\}$$

So our total solution is

$$x = \left\{ \frac{\pi}{6} + 2k\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{6} + 2k\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{2\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{4\pi}{3} + 2k\pi \mid k \in \mathbb{Z} \right\}$$

39. We have:

$$2 \cos 3x = 1$$

or

$$\cos 3x = \frac{1}{2}$$

or

$$3x = \cos^{-1} \frac{1}{2}$$

Now, $\cos \theta = \frac{1}{2}$ when $\theta = \frac{\pi}{3} + 2k\pi$ or $\theta = \frac{5\pi}{3} + 2k\pi$ for any k . So the solutions that fit in the interval desired are

$$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$$

49. We have

$$\sec x - 5 = 0$$

or

$$\frac{1}{\cos x} = 5$$

or

$$\cos x = \frac{1}{5}$$

And a calculator will tell us that

$$x = 78.463^\circ, 281.537^\circ$$