23 January 2005

Homework 1 Solutions

Solutions are given below. Problems that involve graphing or the use of a graphing calculator are not done. These should be used for reference only!

2.1

3. $f(x) = (x-4)^2$

7. Square x, then multiply by three, and then subtract two.

15.

$$\begin{aligned}
g(2) &= \frac{1-2}{1+2} = \frac{-1}{3} \\
g(-2) &= \frac{1-(-2)}{1+(-2)} = -3 \\
g(\frac{1}{2}) &= \frac{1-\frac{1}{2}}{1+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3} \\
g(a) &= \frac{1-a}{1+a} \\
g(a-1) &= \frac{1-(a-1)}{1+(a-1)} = \frac{2-a}{a} \\
g(-1) &= \frac{1-(-1)}{1+(-1)} = \frac{2}{0} = DNE
\end{aligned}$$

45. The function is defined on $-1 \leq x \leq 5$, and the function can be computed at each value, so the domain is $-1 \leq x \leq 5$.

57. If $x < -2$, then $2+x < 0$ so we wouldn't be able to take the square root. Also, if $x = 3$ then we divide by zero. So every other value of x is okay, so the domain is $\{x \mid x \geq -2 \text{ and } x \neq 3\}$.

2.2

3. (a) $f(0) > g(0)$

(b) $f(-3) > g(-3)$

(c) $f(x) = g(x)$ for $x = -2, 2$

9. This is not a function. It fails the vertical line test at $x = -1$.

13. (know the graph!) The domain of $f(x) = x^2 - 4$ is all real numbers.

37. Know the graph!

63. Know the graph!
69. Know the graph!
75. The function can be expressed as follows:

$$f(x) = \begin{cases} 2 & 0 < x \leq 1 \\ 2.2 & 1 < x \leq 1.1 \\ 2.4 & 1.1 < x \leq 1.2 \\ 2.6 & 1.2 < x \leq 1.3 \\ 2.8 & 1.3 < x \leq 1.4 \\ 3.0 & 1.4 < x \leq 1.5 \\ 3.2 & 1.5 < x \leq 1.6 \\ 3.4 & 1.6 < x \leq 1.7 \\ 3.6 & 1.7 < x \leq 1.8 \\ 3.8 & 1.8 < x \leq 1.9 \\ 4.0 & 1.9 < x \leq 2.0 \end{cases}$$

2.3
11. $R = \frac{ki}{Pt}$
13. $y = kx$. If $x = 6$ when $y = 42$, then it implies that $k = 7$.
17. $W = \frac{k}{r^2}$. If $W = 10$ when $r = 6$, then it means that $10 = \frac{k}{36}$, or $k = 360$.
27. (a) $R = \frac{kL}{d^2}$
(b) With the information given, we have $R = 140, L = 1.2$, and $d = 0.005$. So we have

$$140 = \frac{1.2k}{0.005^2}$$

so

$$k = \frac{(140)(0.005)^2}{1.2} = 0.00291\overline{6}$$

(c) With the information given, we have $L = 3$ and $d = 0.008$, so

$$R = \frac{(0.00291\overline{6})(3)}{0.008^2} = 136.71875$$

2.4
23. The function is increasing on $\{x \mid -1 \leq x < 1\}$ and $\{x \mid 2 < x \leq 4\}$. The function is decreasing on $\{x \mid 1 < x < 2\}$.
31. Do it!
33. Do it!

2.5

11. Do it!
35. Start with $y = x^2$, then shift to the left by three, then shift up by five.
41. Do it!
45. Do it!
51. This function is even: $f(-x) = (-x)^{-2} = \frac{1}{(-x)^2} = \frac{1}{x^2} = x^{-2} = f(x)$.

2.6
7.

$$\begin{aligned} y &= x^2 + 6x + 8 \\ &= (x+3)^2 - 1 \end{aligned}$$

So the vertex is $(-3, 1)$. The y-intercept is $(0, -1)$, and the x-intercepts are the roots, which can be found by factoring: $x^2 + 6x + 8 = (x+4)(x+2)$, so the roots are $x = -2$ and $x = -4$.
25. The minimum value is found when $x = \frac{-b}{2a}$. In this case, $a = 1$ and $b = 1$. So $x = \frac{-1}{2}$, so the minimum value is

$$f(\frac{-1}{2}) = (\frac{-1}{2})^2 + (\frac{-1}{2}) + 1 = \frac{3}{4}$$

41. Again the maximum value is obtained when $x = \frac{-b}{2a}$, and in this case $a = -0.4$ and $b = 80$, so $x = \frac{-80}{-0.8} = 100$. The maximum revenue we obtain is

$$f(100) = 80(100) - 0.4(100)^2 = 4000$$

45. Do it!
51. Do it!

2.7
3. $V = w(w/2) = \frac{w^2}{2}$
13. After t hours, the first ship is $15t$ miles south of the port, and the second ship is $20t$ miles east of the port. We can use the Pythagorean Theorem to get the distance between the two:

$$D = \sqrt{(15t)^2 + (20t)^2}$$

23. (b) If x is one of the sides, then the area is $A = x(2400 - 2x)$.
(c) The largest area occurs when $x = \frac{-b}{2a}$, where $a = -2$ and $b = 2400$. So $x = \frac{-2400}{-4} = 600$, so the maximum area is $A = 600(1200) = 720000$.

29. (a) If x is the length of the bottom edge, then πx is needed for the semicircle, and so $30 - (\pi + 1)x$ is left for the other two sides, so each side gets $\frac{30-(\pi+1)x}{2}$. So the total area is

$$A = x(\frac{30 - (\pi + 1)x}{2}) + \frac{\pi(x/2)^2}{2}$$

(b) When multiplied out, we get $A = (\frac{\pi}{8} - \frac{(\pi+1)}{2})x^2 + 15x$, so the maximum comes at

$$x = \frac{-b}{2a} = \frac{-15}{\frac{\pi}{4} - (\pi + 1)}$$

31. Do it!