

Quiz 3 - Math 112, Sections 20-22

Calculators are **not** allowed for this exam. Please mark your final answers clearly. Give exact answers.

1. If $\log_3 \left(\frac{\sqrt[6]{xy^4}}{\sqrt[3]{x^2y^2}} \right) = A \log_3 x + B \log_3 y$, then what are the values of A and B ?
2. Let $f(x) = \ln \sqrt{x}$ and $g(x) = \sqrt{\ln x}$, then what are the domains of f and g ?
3. If $\log_{10} b = \frac{1}{2}$, then compute

$$\log_b 10,000$$

Solutions.

1. $\log_3 \left(\frac{\sqrt[6]{xy^4}}{\sqrt[3]{x^2y^2}} \right) = \log_3(x^{-1/2}) = \frac{-1}{2} \log_3(x)$, so this means that $A = \frac{-1}{2}$ and $B = 0$.
2. For $f(x)$, we can't take the log of anything non-positive, so we need $\sqrt{x} > 0$. This happens when $x > 0$, so the domain of f is $(0, \infty)$. Similarly, for $g(x)$ we can't take the square root of a negative number, so we need $\ln x \geq 0$, so $x \geq 1$, so the domain of g is $[1, \infty)$.
3. Since $\log_{10} b = \frac{1}{2}$, then $10^{\frac{1}{2}} = b$. So if $\log_b 10,000 = x$, then $b^x = 10,000$, or

$$(10^{\frac{1}{2}})^x = 10^4$$

meaning

$$10^{\frac{x}{2}} = 10^4$$

or $\frac{x}{2} = 4$. Hence $x = 8$, so $\log_b 10,000 = 8$.