18 April 2006

Quiz 8 - Math 112, Sections 20-22

1. State the Law of Cosines (include a picture for reference) and show how it reduces to the Pythagorean Theorem.

2. Verify the following trig identities, or give a counterexample if it is not, in fact, an identity.

   (a) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

   (b) $\sin^3 x + \cos^2 x = \tan x$

   (c) $\tan^2 u - \sin^2 u = \tan^2 u \sin^2 u$

Solution for Problem 1. The Law of Cosines is as follows: for a triangle with sides a, b, c and θ the angle opposite c, we have

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

We get the Pythagorean Theorem because in that case we have a right triangle, so the angle opposite of c is 90, and since $\cos 90 = 0$ we are left with

$$c^2 = a^2 + b^2$$

Solution for Problem 2. For the first one,

$$\begin{aligned}(\sin x + \cos x)^2 &= \sin^2 x + 2\sin x\cos x + \cos^2 x \\ &= (\sin^2 x + \cos^2 x) + 2\sin x\cos x \\ &= 1 + 2\sin x\cos x\end{aligned}$$

The second one is false, for example with $x = 0$:

$$\sin^3 0 + \cos^2 0 = 1 \neq \tan 0 = 0$$

The third one is true, as

$$\begin{aligned}\tan^2 u - \sin^2 u &= \left(\frac{\sin^2 u}{\cos^2 u} - \sin^2 u\right) \\ &= \sin^2 u\left(\frac{1}{\cos^2 u} + 1\right) \\ &= \sin^2 u(\sec^2 u + 1) \\ &= \sin^2 u \tan^2 u\end{aligned}$$

And we're done!