

25 April 2006

Quiz 9 - Math 112, Sections 20-22

Calculators are **NOT** allowed on this exam. The back of this sheet may help.

1. Evaluate the following.

(a) $\sin\left(\frac{\pi}{12}\right)$

(b) $\cos\left(\frac{7\pi}{12}\right)$

2. Prove the following identity:

$$\frac{\sin 4x}{\sin x} = 4 \cos x \cos 2x$$

Solution for Problem 1. For the first one, since $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ then we have

$$\begin{aligned} \sin \frac{\pi}{12} &= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{1}{2} \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

For the second one, since $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, then we have

$$\begin{aligned} \cos \frac{7\pi}{12} &= \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) \\ &= \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{1}{2} \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

Solution to Problem 2. We have

$$\begin{aligned} \frac{\sin 4x}{\sin x} &= \frac{2 \sin 2x \cos 2x}{\sin x} \\ &= \frac{2(2 \sin x \cos x) \cos 2x}{\sin x} \\ &= 4 \cos x \cos 2x \end{aligned}$$

And we're done!