16 January 2007

Calculus I Warm-up

Instructions: You may turn in solutions at most once every two weeks. This assignment counts for two homework grades. **All solutions must be exact.** More detailed instructions will be on the webpage.

1. Let $f(x) = 6x^2 + 2x + 11$.

   (a) At what value of x does f reach its minimum?

   (b) What is the minimum value of f?

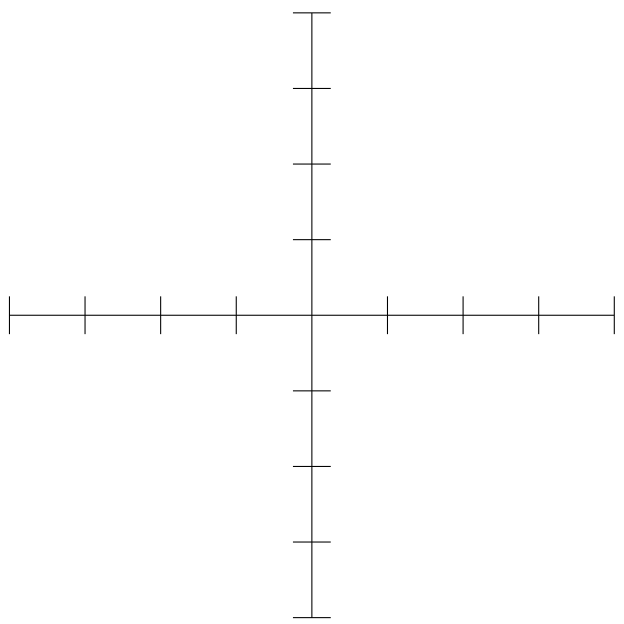
   (c) What is the vertex of f?

2. Plot the following three functions on the axes given below. Label each function clearly! Each tick mark on the axes represents one unit.

   (a) $f(x) = x^2$

   (b) $g(x) = (x + 5)^2$

   (c) $h(x) = (x + 5)^2 - 2$



3. Eight functions with their domains are given. Circle Y if the function is 1-1 on the specified domain, and N otherwise. $\mathbb{R}$ denotes the real numbers.

	function	domain		
(a)	$f(x) = x$	$\mathbb{R}$	Y	N
(b)	$f(x) = \lvert x \rvert$	$\mathbb{R}$	Y	N
(c)	$f(x) = x^2 + 3x + 9$	$\mathbb{R}$	Y	N
(d)	$f(x) = x^2$	$\{x \in \mathbb{R} \mid x \geq 0\}$	Y	N
(e)	$f(x) = x^2$	$\{x \in \mathbb{R} \mid x > 5\}$	Y	N
(f)	$f(x) = e^x$	$\mathbb{R}$	Y	N
(g)	$f(x) = \begin{cases} 3 - x & \text{if } x < 0 \\ x^2 - 3 & \text{otherwise} \end{cases}$	$\mathbb{R}$	Y	N
(h)	$f(x) = \frac{1}{x+9}$	$\{x \in \mathbb{R} \mid x \neq -9\}$	Y	N

4. Let $f(x) = C \cdot a^x$. If $(0, 4)$ and $(4, 64)$ are on the graph of f, then what is C and a?

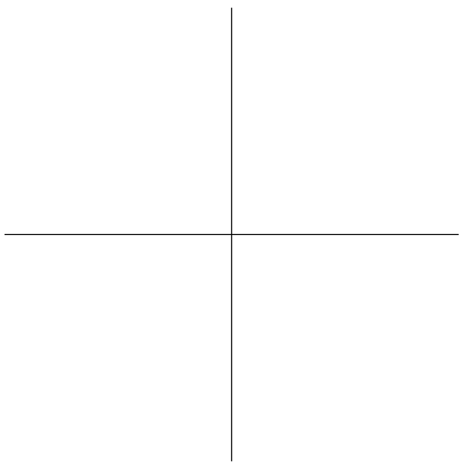
5. If $\log_3 \left(\frac{\sqrt[6]{x^2 y}}{\sqrt[5]{x^3 y^4}} \right) = A \log_3 x + B \log_3 y$, then what are the values of A and B?

6. Let $f(x) = \ln \sqrt[3]{x}$ and $g(x) = \sqrt[3]{\ln x}$, then what are the domains of f and g?

7. If $\log_2 b = \frac{2}{3}$, then compute
$$\log_b 32$$

8. The following table has labels and real numbers. On the unit circle below, mark and label the terminal point detemined by the number on the circle, give the reference number, and give the coordiantes of the terminal point. The first one is done for you as an example.

Label	**Value**	**Reference Number**	**Coordinates**
A	0	0	(1,0)
B	$\frac{\pi}{4}$		
C	$\frac{5\pi}{4}$		
D	$\frac{11\pi}{6}$		
E	$\frac{2\pi}{3}$		
F	7π		
G	$\frac{10\pi}{4}$		
H	$\frac{4\pi}{3}$		
I	$\frac{7\pi}{4}$		



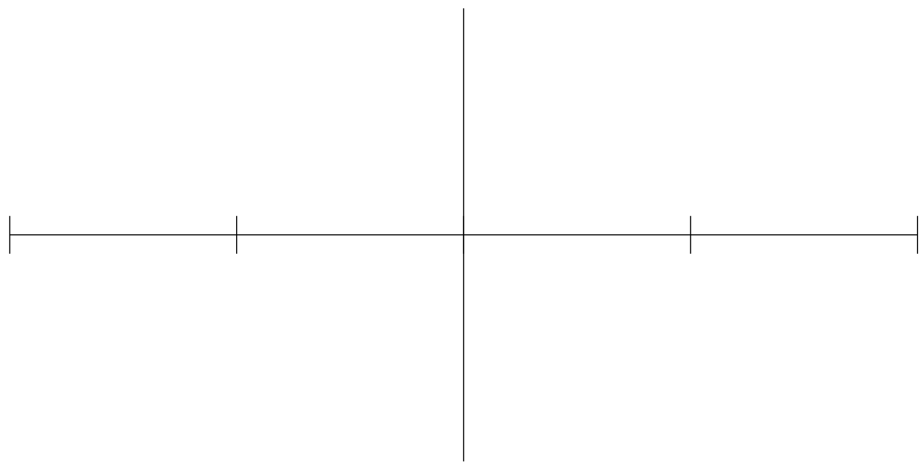
9. Evaluate the following.

   (a) $\sin(\frac{\pi}{3})$
   (b) $\cos(\frac{7\pi}{6})$
   (c) $\tan(\frac{3\pi}{4})$
   (d) $\sec(\frac{\pi}{2})$
   (e) $\csc(5\pi)$
   (f) $\cot(\frac{11\pi}{6})$

10. Consider the function

$$f(x) = 3\sin(\frac{\pi}{2}x - 3\pi)$$

Graph the function on the axes below. **Note:** The distance between each of the hash marks on the x-axis must represent one period, so label correctly!



11. Evaluate the following.

    (a) $\tan(\frac{2\pi}{3})$
    (b) $\sin(\frac{\pi}{6})$
    (c) $\cos(\frac{5\pi}{4})$
    (d) $\csc(\frac{3\pi}{2})$
    (e) $\cot(8\pi)$
    (f) $\sec(\frac{7\pi}{6})$

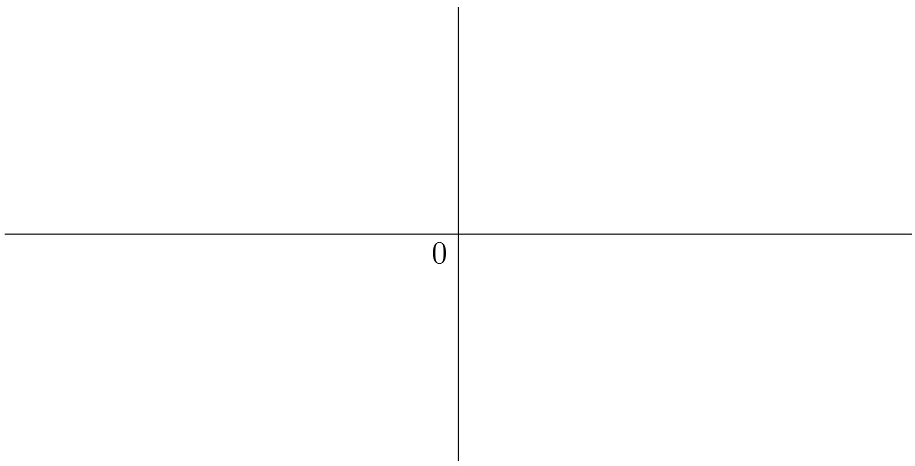
12. Consider the function

$$f(x) = 3\tan(\frac{\pi}{6}x - 3\pi)$$

State the following values:

Period: Phase Shift:

Graph the function on the axes below.



13. Evaluate the following.

   (a) $\cos(\frac{2\pi}{3})$
   (b) $\tan(\frac{\pi}{6})$
   (c) $\csc(\frac{5\pi}{4})$
   (d) $\sin(\frac{5\pi}{2})$
   (e) $\sec(6\pi)$
   (f) $\cot(\frac{7\pi}{6})$

14. Find the area of an equilateral triangle with side length 2.

15. Given the following information, sketch the triangle and solve the triangle. Be clear.

$$\angle A = 64^\circ,\ \angle B = 33^\circ,\ c = 25$$

16. State the Law of Cosines (include a picture for reference) and show how it reduces to the Pythagorean Theorem.

17. Verify the following trig identities, or give a counterexample if it is not, in fact, an identity.

   (a) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$
   (b) $\sin^3 x + \cos^2 x = \tan x$
   (c) $\tan^2 u - \sin^2 u = \tan^2 u \sin^2 u$

18. Evaluate the following.

   (a) $\sin(\frac{\pi}{12})$
   (b) $\cos(\frac{7\pi}{12})$

19. Prove the following identity:

$$\frac{\sin 4x}{\sin x} = 4\cos x \cos 2x$$