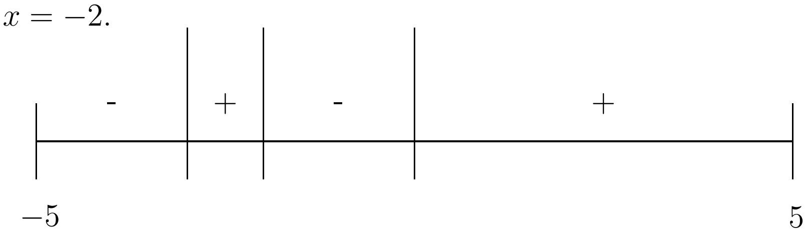
28 March 2007

Quiz 8: Math 135, Sections 1-3

Consider the function $f(x) = (x^3 + 3x^2)^3$ in the interval $[-5, 5]$.

1. Find all the first-order critical points, and mark where the function is increasing and decreasing on the line below. Also mark whether each critical point is a relative min or a relative max.

We have $f'(x) = 3(x^3 + 3x^2)^2(3x^2 + 6x) = 9x^3(x + 3)(x + 2)$. Hence, we have critical points at $x = 0, -2, -3$. By considering specific values, we see that we are increasing on $(-3, -2)$ and $(0, 5]$ and are decreasing on $[-5, -3)$ and $(-2, 0)$. Hence, we have relative mins at $x = -3$ and $x = 0$ and a relative max at $x = -2$.

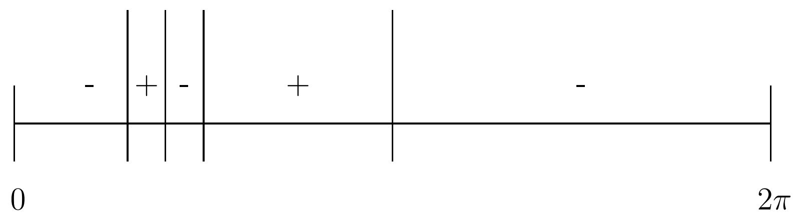


2. On the line below, find all the second-order critical points and mark where the function is concave up and concave down. Also mark inflection points, if any.

We have

$$\begin{aligned}
f''(x) &= 3[(x^3 + 3x^2)^2(6x + 6) + 2(x^3 + 3x^2)(3x^2 + 6x)(3x^2 + 6x)] \\
&= 3(x^3 + 3x^2)[x^2(x + 3)(6x + 6) + 2x^2(3x + 6)^2] \\
&= 18x^2(x^3 + 3x^2)[(x + 3)(x + 1) + 3(x + 2)^2] \\
&= 18x^2(x^3 + 3x^2)[4x^2 + 16x + 15] \\
&= 18x^4(x + 3)(2x + 5)(2x + 3)
\end{aligned}$$

So our critical points are at $x = 0, -3, -5/2, -1/2$. Looking at signs, we have that the second derivative is negative on $[-5, -3), (-5/2, -1/2), (0, 5]$ and positive on $(-3, -5/2), (-1/2, 0)$. Therefore, all four critical points are inflection points.



3. On the back of this page, sketch a graph of the function.

This wasn't graded on the quiz (question 2 wasn't really, either). You can use your graphing calculator or a computer program to look at the graph.