31 January 2007

Quiz 3: Math 135, Sections 20-22

Evaluate the following limits - if it does not exist, explain why.

1. $\lim_{x\to 3} x^2 + 3x + 5$
2. $\lim_{x\to 2} \frac{x^2+2x-8}{x-2}$
3. $\lim_{x\to 0} \frac{1}{x}$
4. $\lim_{x\to 0} \frac{1-\cos x}{\sin x}$
5. $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ -x & \text{if } x \text{ is irrational} \end{cases}$

Solutions.

1. $\lim_{x\to 3} x^2+3x+5 = \left(\lim_{x\to 3} x\right)^2+3\left(\lim_{x\to 3} x\right)+\lim_{x\to 3} 5 = 3^3+3\cdot 3+5 = 23.$

2. Since, when $x \neq 2$, we have

$$\frac{x^2+2x-8}{x-2} = \frac{(x-2)(x+4)}{x-2} = x+4$$

then the limit is $2+4=6$.

3. This limit does not exist because we shoot off to negative infinity from the left, and we shoot off to positive infinity from the right. A simple plot of the function would show this.

4. $\lim_{x\to 0} \frac{1-\cos x}{\sin x} = \lim_{x\to 0} \frac{1-\cos x}{\sin x}\cdot\frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{1-\cos^2 x}{\sin x(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{\sin x(1+\cos x)} = \lim_{x\to 0} \frac{\sin x}{1+\cos x} = \frac{\sin 0}{1+\cos 0} = \frac{0}{2} = 0$

5. This function is bounded from above by the function $f_1(x) = |x|$, and it is bounded from below by $f_2(x) = -|x|$. Since $\lim_{x\to 0} f_1(x) = \lim_{x\to 0} f_2(x) = 0$, then by the squeeze rule we have $\lim_{x\to 0} f(x) = 0$.