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Quiz 3 Solutions: Math 135, Sections 1-3

1. Show that the given equation has at least one solution on the indicated interval:

$$\cos x - \sin x = x \qquad \text{on } (0, \frac{\pi}{2})$$

2. Find the given limit, or say why it does not exist:

$$\lim_{x \to 0^+} e^{\frac{1}{x}}$$

3. For what value of A will f be continuous?

$$f(x) := \begin{cases} 3x - 2 & \text{if } x < 1 \\ Ax^3 + Ax + 7 & \text{otherwise} \end{cases}$$

Solutions.

1. Finding a solution to the equation

$$\cos x - \sin x = x$$

is equivalent to finding a root of the equation

$$\cos x - \sin x - x = 0$$

Note, at $x = 0$:

$$\cos 0 - \sin 0 - 0 = 1 > 0$$

and at $x = \frac{\pi}{2}$:

$$\cos \frac{\pi}{2} - \sin \frac{\pi}{2} - \frac{\pi}{2} = -1 - \frac{\pi}{2} < 0$$

So, by the Intermediate Value Theorem, there is some x between 0 and $\frac{\pi}{2}$ such that $\cos x - \sin x - x = 0$, and we're done.

2. As $x \to 0$ from the right, $\frac{1}{x} \to +\infty$, hence $e^{\frac{1}{x}} \to +\infty$ also.

3. We need $\lim_{x \to 1} = f(1)$, as $x = 1$ is our only suspicious point. Note that

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 3x - 2 = 1$$

and

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} Ax^3 + Ax + 7 = 2A + 7$$

so we need $2A + 7 = 1$, meaning $A = -3$.