♡ 14 February 2007 ♡

Quiz 4: Math 135, Sections 1-3

Let $f(x) = x^2 \ln(x) - 3x^3 + 4$.

1. What is $f'(x)$?
2. What is the **exact** equation of the tangent line to f at $x = e$?
3. What is the **exact** equation of the normal line to f at $x = e^2$?

Extra Credit Questions (2 points each):

(E1) Recall the derivative of a function is defined by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Use this limit definition to find the derivative of $3x^2 + 6x + 4$.

(E2) What function g satisfies $g'(x) = 3x^2 + 6x + 4$? (This is called the *antiderivative.*)

Solutions (non-EC problems).

1. $f'(x) = (2x\ln(x) + x^2 \frac{1}{x}) - 9x^2 = 2x\ln(x) + x - 9x^2$.

2. The slope of the tangent line at $x = e$ is $f'(e) = 2e\ln(e) + e - 9e^2 = 3e - 9e^2$. The point at which this line passes through is $(e, f(e)) = (e, e^2 - 3e^3 + 4)$, and hence the line we seek is

$$y - (e^2 - 3e^3 + 4) = (3e - 9e^2)(x - e)$$

3. The slope of the *tangent* line at $x = e^2$ is $f'(e^2) = 2e^2\ln(e^2) + e^2 - 9e^4 = 5e^2 - 9e^4$. Hence, the slope of the normal line is $\frac{-1}{5e^2 - 9e^4}$. The point at which this line passes through is $(e^2, f(e^2)) = (e^2, e^4\ln(e^2) - 3e^6 + 4) = (e^2, 2e^4 - 3e^6 + 4)$, and so the line we seek is

$$y - (2e^4 - 3e^6 + 4) = \frac{-1}{5e^2 - 9e^4}(x - e^2)$$