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Quiz 5: Math 135, Sections 1-3

Find the derivatives of the following functions:

1. $f(x) = \frac{1}{x^2}$
2. $g(x) = \sin \frac{1}{x^2}$
3. $h(x) = \sqrt{\sin \frac{1}{x^2}}$

Solutions.

1. $f(x) = x^{-2}$ so $f'(x) = -2x^{-3} = \frac{-2}{x^3}$.
2. If we let $u = \frac{1}{x^2}$, then $g(x) = \sin u$ and
$$\frac{du}{dx} = \frac{-2}{x^3}$$
from problem 1. Then
$$\frac{dg}{dx} = \frac{dg}{du}\frac{du}{dx} = \cos u \frac{-2}{x^3} = \cos\left(\frac{1}{x^2}\right)\frac{-2}{x^3}$$
3. If we let $u = \sin \frac{1}{x^2}$ then $h(x) = \sqrt{u}$ and
$$\frac{du}{dx} = \cos\left(\frac{1}{x^2}\right)\frac{-2}{x^3}$$
from problem 2. Then
$$\frac{dh}{dx} = \frac{dh}{du}\frac{du}{dx} = \frac{1}{2\sqrt{u}}\cos\left(\frac{1}{x^2}\right)\frac{-2}{x^3} = \frac{1}{2\sqrt{\sin \frac{1}{x^2}}}\cos\left(\frac{1}{x^2}\right)\frac{-2}{x^3}$$