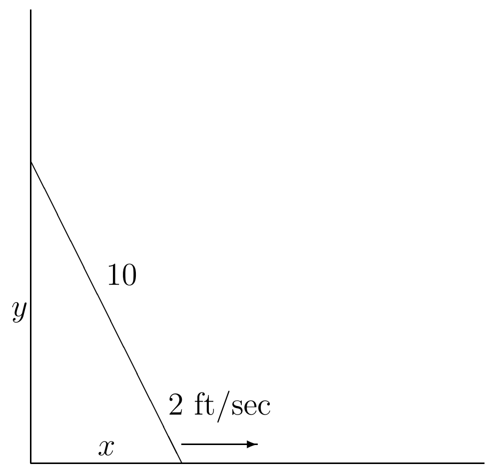
7 March 2007

Quiz 6: Math 135, Sections 1-3

A 10-foot ladder is vertical against a wall at time $t = 0$. At that time, the bottom of the ladder is pulled away from the wall at a rate of 2 feet per second. We want to consider how fast the top of the ladder will move down as the base is pulled away.

1. Draw a picture of the situation and label appropriate variables.

At some moment t, the scenario is as follows:



2. Let $v_{\downarrow}(t)$ denote the speed that the top of the ladder is moving down at time t. Give an expression for $v_{\downarrow}(t)$ and specify its domain. (Hint: speed is the change in position.)
We always have that

$$x^2 + y^2 = 100$$

and so, taking the derivative with respect to t we obtain

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$v_{\downarrow}(t)$ simply represents $\frac{dy}{dt}$ and so we have

$$v_{\downarrow}(t) = \frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y}$$

3. What is

$$\lim_{t\to 5} v_{\downarrow}(t) \ \ ?$$

What does that mean, in real-life terms?

When $t \to 5$, we have that $x \to 10$ and $y \to 0$. $\frac{dx}{dt}$ is always 2, so we can conclude that

$$\lim_{t\to 5} v_{\downarrow}(t) = \lim_{t\to 5} \frac{dy}{dt} = \lim_{y\to 0} \frac{20}{y} = +\infty$$

Therefore, the speed of the top of the ladder gets infinitely high as it approaches the ground. Obviously this cannot happen and we would be unable to move the bottom of the ladder at a constant rate of 2 ft/sec the whole time.

Officially I should have asked for

$$\lim_{t\to 5^-} v_{\downarrow}(t)$$

as that would produce

$$\lim_{y\to 0^+} \frac{20}{y}$$

but there was no confusion.