

21 March 2007

Quiz 7: Math 135, Sections 1-3

1. Using a linear approximation from $x = 8$, give an estimation for the value of $\sqrt[3]{7.9}$.

We estimate using the function $f(x) = \sqrt[3]{x}$. We have $f'(x) = \frac{1}{3}x^{-2/3}$ and so the slope of the tangent line at $x = 8$ is

$$y - 2 = \frac{1}{12}(x - 8)$$

Therefore,

$$\sqrt[3]{7.9} \approx \frac{1}{12}(7.9 - 8) + 2 = 1.991666667$$

(For reference, a calculator will give you 1.991631701)

2. Find the absolute maximum and the absolute minimum, if they exist, of the function

$$f(x) = x^3 - 9x^2 + 24x + 2$$

on the interval $[3, 6]$.

We have $f'(x) = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = 3(x - 4)(x - 2)$ so $f'(x) = 0$ when $x = 2$ and $x = 4$. We only need to consider 4, as 2 is outside of the interval. Our critical points are 3, 4, and 6. Therefore,

x	$f(x)$
3	20
4	18
6	38

Hence, our absolute minimum is 18 (at $x = 4$) and our absolute maximum is 38 (at $x = 6$).