

4 April 2007

Quiz 9: Math 135, Sections 1-3

Note. Calculators are **NOT** allowed on this quiz. Problems 2 and 3 are all-or-nothing.

1. (4 points) Farmer Raff has 960 feet of fence to build a pen. He has four different types of animals: sheep, pigs, cows, and horses. He wants to make a rectangular pen with his fence that is subdivided into four smaller equally-sized sub-pens as shown here:



What is the largest total area I can achieve of this glorious pen? Remember your units!

2. (2 points) Evaluate the following limit.

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 5x + 4}{x^2 - 6x + 4,000,000,000}$$

3. Let $f(x) = \frac{x^2+2x-8}{x^2+8x-20}$.
 - (a) (2 points) What are the horizontal asymptotes?
 - (b) (2 points) What are the vertical asymptotes?

Solutions.

1. Let x denote a horizontal section of the fence, and let y denote a vertical section. We want to maximize

$$A = xy$$

and since we only have 960 feet of fence, it means that

$$3x + 3y = 960$$

Solving for y , we get $y = 320 - x$ and so our equation is now

$$A = x(320 - x) = 320x - x^2$$

Therefore,

$$A' = 320 - 2x$$

So our area is maximized when $x = 160$, which makes $y = 160$, and the total area would be 25,600.

2. When we divide the top and the bottom by x^2 , we get

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 5x + 4}{x^2 - 6x + 4,000,000,000} = \lim_{x \rightarrow \infty} \frac{3 + \frac{5}{x} + \frac{4}{x^2}}{1 - \frac{6}{x} + \frac{4,000,000,000}{x^2}} = 3$$

3. We get horizontal asymptotes by evaluating

$$\lim_{x \rightarrow \infty} \frac{x^2 + 2x - 8}{x^2 + 8x - 20}$$

and

$$\lim_{x \rightarrow -\infty} \frac{x^2 + 2x - 8}{x^2 + 8x - 20}$$

The two limits are the same, and they both equal one.

For the vertical asymptotes, note that

$$\frac{x^2 + 2x - 8}{x^2 + 8x - 20} = \frac{(x + 4)(x - 2)}{(x + 10)(x - 2)} = \frac{x + 4}{x + 10}$$

Therefore, the only vertical asymptote is at $x = -10$. There is no vertical asymptote at $x = 2$.