25 June 2007

Quiz 10: Math 135, Section C7

1. Find the equation of the tangent line of the graph $f(x) = 3\sin(x^2)e^x$ at the point $x = 1$.

2. Find $d(x^5 + \sqrt{x^2+5})$

3. Approximate $\sqrt[3]{7.99}$.

4. Approximate $2^{2.01}$.
(*Hint:* $2 = e^{\ln 2}$.)

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1. We have, by the product rule and the chain rule,

$$f'(x) = 3[\cos(x^2)2x(e^x) + \sin(x^2)e^x] = 3e^x(2x\cos(x^2) + \sin(x^2))$$

and so the slope of our tangent line is

$$f'(1) = 3e(2\cos(1) + \sin(1))$$

Our y-coordinate is

$$f(1) = 3e\sin(1)$$

so the equation of our tangent line is

$$y - 3e\sin(1) = 3e(2\cos(1) + \sin(1))(x - 1)$$

2. We have

$$\frac{d}{dx}\left(x^5 + \sqrt{x^2+5}\right) = 5x^4 + \frac{1}{2\sqrt{x^2+5}}(2x)$$

and so

$$d(x^5 + \sqrt{x^2+5}) = \left(5x^4 + \frac{1}{2\sqrt{x^2+5}}(2x)\right)\,dx$$

3. We will find the tangent line to $f(x) = \sqrt[3]{x}$ at $x = 8$. We have

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

So the slope of our tangent line is

$$f'(8) = \frac{1}{3 \cdot 8^{2/3}} = \frac{1}{12}$$

Our y-coordinate is

$$f(8) = \sqrt[3]{8} = 2$$

So our line is

$$y - 2 = \frac{1}{12}(x - 8)$$

so our approximation is

$$y = \frac{1}{12}(7.99 - 8) + 2$$

4. We will find the tangent line to $f(x) = 2^x$ at $x = 2$. We have

$$f'(x) = 2^x \ln 2$$

So the slope of our tangent line is

$$f'(2) = 4 \ln 2$$

Our y-coordinate is

$$f(2) = 4$$

so our line is

$$y - 4 = 4 \ln 2(x - 2)$$

and our approximation is

$$y = 4 \ln 2(2.01 - 2) + 4$$