27 June 2007

Quiz 11: Math 135, Section C7

Let $f(x) = 3x^5 - 20x^3$ on the interval $[-1, 2]$.

1. Find the critical numbers for f.

2. Classify each critical number as an absolute maximum, an absolute minimum, or neither.

Let $g(x) = \frac{x}{e^x}$ on the interval $[-2, 2]$.

3. Find the critical numbers for g.

4. Classify each critical number as an absolute maximum, an absolute minimum, or neither.

Extra credit (2 points): Give an example of a specific function defined on some specific interval that has neither an absolute maximum nor an absolute minimum on that interval.

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1. Critical numbers are where $f'(x) = 0$ or $f'(x)$ does not exist. We have

$$f'(x) = 15x^4 - 60x^2 = 15x^2(x^2 - 4) = 15x^2(x - 2)(x + 2)$$

So our critical points are $-2, 0, 2$. We don't care about -2, though, since it is not in our interval.

2. We check the critical points and our endpoints, and we have the table

x	$f(x)$	
-1	17	(abs. max)
0	0	(neither)
2	-64	(abs. min)

3. We have

$$g'(x) = \frac{e^x - xe^x}{e^{2x}}$$

So $g'(x) = 0$ when

$$e^x - xe^x = e^x(1 - x) = 0$$

so this happens when $x = 1$.

4. We check $x = 1$ plus the endpoints, and we have the table

x	$f(x)$	
-2	$-2e^2$	(abs. min)
1	$\frac{1}{e}$	(abs. max)
2	$\frac{2}{e^2}$	(neither)