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Quiz 13: Math 135, Section C7

1. Compute the following limit:

$$\lim_{x\to\infty} \frac{3x}{\sqrt{x^2+1000}}$$

For the next five problems, let

$$f(x) = \frac{3x+5}{7-x}$$

2. What are the vertical asymptote(s) of this function?

3. What are the horizontal asymptote(s) of this function?

4. When is this function increasing? Decreasing?

5. When is this function concave up? Concave down?

6. On the axes on the back, sketch this function.

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1. If we divide the top and the bottom by $\frac{1}{x}$, we get

$$\lim_{x\to\infty} \frac{3x}{\sqrt{x^2+1000}} \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{3}{\sqrt{1+\frac{1000}{x^2}}} = \frac{3}{\sqrt{1+0}} = 3$$

2. The vertical asymptotes occur when the denominator is zero. Solving $7-x=0$ we get that $x=7$.

3. The horizontal asymptotes occur at the limits to $-\infty$ and ∞. In the same manner as solving problem 1, we see that the limits both equal -3.

4. We need to find the derivative:

$$f'(x) = \frac{(3)(7-x)-(3x+5)(-1)}{(7-x)^2} = \frac{26}{(7-x)^2}$$

Since $(7-x)^2$ is always non-negative, we see that this function is always increasing (where it is defined).

5. We need the second derivative:

$$f''(x) = \frac{(0)(7-x)^2 - (26)2(7-x)(-1)}{(7-x)^4} = \frac{52}{(7-x)^3}$$

Since $(7-x)^3 > 0$ when $x < 7$ and $(7-x)^3 < 0$ when $x > 7$, then we are concave up when $x < 7$ and concave down when $x > 7$.