6 June 2007

Quiz 3: Math 135, Section C7

1. Consider the function

$$g(t) = \begin{cases} 3t+2 & \text{if } t \leq 1 \\ 5 & \text{if } 1 < t \leq 3 \\ 3t^2 - 1 & \text{if } t > 3 \end{cases}$$

What are the suspicious points of g and what are the points of discontinuity of g?

2. Show that the equation

$$\cos x - \sin x = x$$

has a solution on the interval $(0, \frac{\pi}{2})$.

3. Consider the function

$$f(x) = \begin{cases} x^2 + bx + 1 & \text{if } x < 5 \\ 8 & \text{if } x = 5 \\ ax + 3 & \text{if } x > 5 \end{cases}$$

What values of a and b will make f continuous?

Solutions

1. The suspicious points of g are $t = 1$ and $t = 3$, as the function changes definition at those two points. Note that

$$\lim_{t \to 1^-} g(t) = \lim_{t \to 1^-} 3t + 2 = 5$$

and

$$\lim_{t \to 1^+} g(t) = \lim_{t \to 1^+} 5 = 5$$

so

$$\lim_{t \to 1} g(t) = 5 = g(1)$$

and so we are continuous at 1. However, for $t = 3$:

$$\lim_{t\to 3^-} g(t) = \lim_{t\to 3^-} 5 = 5$$

and

$$\lim_{t\to 3^+} g(t) = \lim_{t\to 3^+} 3t^2 - 1 = 26$$

so

$$\lim_{t\to 3} g(t) \text{ does not exist.}$$

so g is not continuous at $t = 3$.

2. If we can show that $f(x) = \cos x - \sin x - x$ has a zero on $(0, \frac{\pi}{2})$, then we'd be done. Note:

$$f(0) = \cos 0 - \sin 0 - 0 = 1 - 0 - 0 = 1(> 0)$$

and

$$f(\frac{\pi}{2}) = \cos\frac{\pi}{2} - \sin\frac{\pi}{2} - \frac{\pi}{2} = -1 - \frac{\pi}{2}(< 0)$$

f is continuous, so we satisfy all the conditions of the IVT. Hence, there is a solution in the interval $(0, \frac{\pi}{2})$.

3. We need

$$\lim_{x\to 5^-} f(x) = f(5) = 8$$

and

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} x^2 + bx + 1 = 5b + 26$$

so we need $5b + 26 = 8$, or $b = -\frac{18}{5}$. Similarly, we need

$$\lim_{x\to 5^+} f(x) = f(5) = 8$$

and

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} ax + 3 = 5a + 3$$

so we need $5a + 3 = 8$, or $a = 1$.