8 June 2007

Quiz 4: Math 135, Section C7

Note: Calculators are *not* allowed for this quiz.

1. Simplify the following expressions:

$$\log_2 4 + \log_3 \frac{1}{9}$$
$$e^{5\ln 2}$$
$$\exp(\log_{e^2} 25)$$

2. Solve the following equation:

$$\log_3 x + \log_3(2x+1) = 1$$

3. Compute the following limit:

$$\lim_{x\to 0^+} x^2 e^{-x}$$

Solutions

1.
$$\log_2 4 + \log_3 \frac{1}{9} = 2 + (-2) = 0$$
$$e^{5\ln 2} = (e^{\ln 2})^5 = 2^5 = 32$$

By using change-of-base, we know that

$$\log_{e^2} 25 = \frac{\ln 25}{\ln e^2} = \frac{\ln 25}{2}$$

so

$$\exp(\log_{e^2} 25) = \exp\left(\frac{\ln 25}{2}\right) = \exp(\ln 25)^{\frac{1}{2}} = 25^{\frac{1}{2}} = 5$$

2. We have

$$\begin{aligned} \log_3 x + \log_3(2x+1) &= 1 \\ \log_3(x(2x+1)) &= 1 \\ x(2x+1) &= 3 \\ 2x^2 + x - 3 &= 0 \\ (2x+3)(x-1) &= 0 \end{aligned}$$

so $x = -\frac{3}{2}$ and $x = 1$ are potential solutions. However, $x = -\frac{3}{2}$ is an invalid solution, so $x = 1$ is our only solution.

3.

$$\lim_{x \to 0^+} x^2 e^{-x} = 0^2 \cdot e^0 = 0 \cdot 1 = 0$$