

11 June 2007

Quiz 5: Math 135, Section C7

Recall that the *difference quotient* of a function f is given by

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

1. Find the difference quotient for the function $g(x) = 3x^2$.
 2. Find the derivative $g'(x)$ by taking the limit of the difference quotient as $\Delta x \rightarrow 0$.
 3. What is the equation of the tangent line of $g(x)$ at the point $x = 3$?
 4. On the back of this page there is a set of axes. Draw the graph $f(x) = x^2 - x$ on the axes, and find out the coordinates of the point where $f'(x) = 0$. What is happening to the graph at that point?
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1. The difference quotient is

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{3(x + \Delta x)^2 - 3x^2}{\Delta x}$$

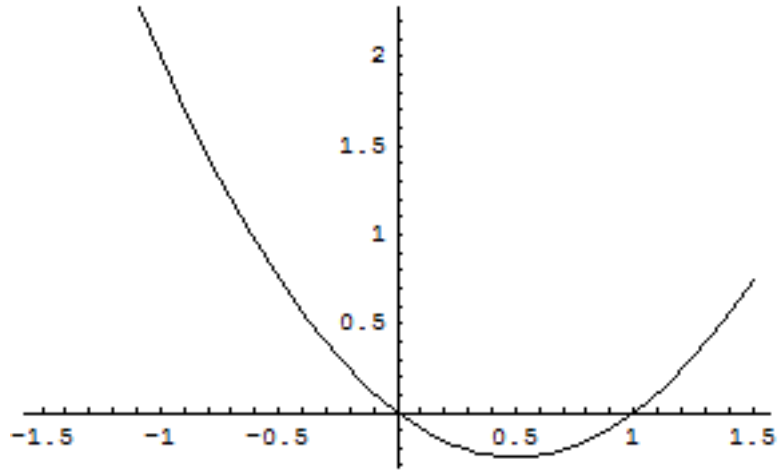
2. We compute the limit:

$$\begin{aligned} \lim_{\Delta x \rightarrow 0} \frac{3(x + \Delta x)^2 - 3x^2}{\Delta x} &= \lim_{\Delta x \rightarrow 0} \frac{3(x^2 + 2x\Delta x + \Delta x^2) - 3x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{3x^2 + 6x\Delta x + 3\Delta x^2 - 3x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{6x\Delta x + 3\Delta x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(6x + 3\Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 6x + 3\Delta x \\ &= 6x \end{aligned}$$

3. At $x = 3$, the slope of the tangent line is $f'(3) = 6(3) = 18$. The y -value of the point involved is $f(3) = 3(3)^2 = 27$. Therefore, the equation of the tangent line is

$$y - 27 = 18(x - 3)$$

4.



The derivative is equal to 0 at the point $(.5,-.25)$. At this point, the function has stopped decreasing and is about to start increasing.