**Trigonometric Values**

| | sin | cos | tan |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ |
| $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 |
| $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| $\frac{\pi}{2}$ | 1 | 0 | DNE |

**Trigonometric Identities**

$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$
$$\sin(s-t) = \sin s \cos t - \cos s \sin t$$
$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$
$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$
$$\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$
$$\tan(s-t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$
$$\sin \frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}}$$
$$\cos \frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$
$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u}$$
$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$$
$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$
$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$
$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

**Table of Integrals**

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$
$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$
$$\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx$$
$$\int \sec^n x \, dx = \frac{sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$
$$\int \tan x \, dx = -\ln|\cos x| + C$$
$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

**Integral Approximations**

Midpoint Rule

$$\int_a^b f(x) \, dx \approx \Delta x[f(x_1^*) + f(x_2^*) + \cdots + f(x_N^*)]$$
$$error(M_N) \le \frac{K_2(b-a)^3}{24N^2}$$

Trapezoid Rule

$$\int_a^b f(x) \, dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{N-1}) + f(x_N)]$$
$$error(T_N) \le \frac{K_2(b-a)^3}{12N^2}$$

Simpson's Rule

$$\int_a^b f(x) \, dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)]$$
$$error(S_N) \le \frac{K_4(b-a)^5}{180N^4}$$

**Areas and Volumes**

| | |
|---|---|
| Area of circle | $\pi r^2$ |
| Volume of sphere | $\frac{4}{3}\pi r^3$ |
| Volume of cone | $\frac{1}{3}\pi r^2 h$ |

**Mean Value Theorem**

If $f(x)$ is continuous on $[a,b]$, then there is some $c$ in the interval $[a,b]$ so that

$$f(c) = \frac{1}{b-a} \int_a^b f(x) \, dx$$

**Arc Length**

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

**Surface Area**

$$S = 2\pi \int_1^b f(x)\sqrt{1 + f'(x)^2}\, dx$$

**Taylor Polynomials**

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$|T_n(x) - f(x)| \le K_{n+1}\frac{|x-a|^{n+1}}{(n+1)!}$$

**Geometric Series**

$$1 + r + r^2 + \cdots = \frac{1}{1-r} \qquad (|r| < 1)$$

**Comparison Test**
Assume that there exists $M > 0$ such that $0 \le a_n \le b_n$ for $n \ge M$.

If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.
If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ also diverges.

**Limit Comparison Test**
Let $\{a_n\}$ and $\{b_n\}$ be *positive* sequences. Assume that the following limit exists:

$$L = \lim_{n\to\infty} \frac{a_n}{b_n}$$

- If $L > 0$, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} b_n$ converges.
- If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

**Leibniz Test**
Let $\{a_n\}$ be a decreasing positive sequence that converges to 0:

$$a_1 \ge a_2 \ge a_3 \ge a_4 \ge \cdots \ge 0. \qquad \lim_{n\to\infty} a_n = 0$$

Then the following alternating series converges:

$$S = \sum_{n=1}^{\infty} (-1)^{n-1} a_n = a_1 - a_2 + a_3 - a_4 + \cdots$$

Furthermore, $0 \le S \le a_1$ and $S_{2N} \le S \le S_{2N+1}$ for all $N$

**Ratio Test**
Assume the following limit exists:

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

(i) If $\rho < 1$, then $\sum a_n$ converges absolutely.
(ii) If $\rho > 1$, then $\sum a_n$ diverges.
(iii) If $\rho = 1$, the Ratio Test is inconclusive (the series may converge or diverge).

**Root Test**
Assume the following limit exists:

$$\rho = \lim_{n\to\infty} \sqrt[n]{a_n}$$

(i) If $\rho < 1$, then $\sum a_n$ converges absolutely.
(ii) If $\rho > 1$, then $\sum a_n$ diverges.
(iii) If $\rho = 1$, the Root Test is inconclusive (the series may converge or diverge).

**Taylor Series**

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

**Common Maclaurin Series**

| Function $f(x)$ | Maclaurin Series |
|---|---|
| $e^x$ | $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ |
| $\sin x$ | $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ |
| $\cos x$ | $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ |
| $\frac{1}{1-x}$ | $\sum_{n=0}^{\infty} x^n$ |
| $\ln(1+x)$ | $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$ |
| $(1+x)^a$ | $\sum_{n=0}^{\infty} \binom{a}{n} x^n$ |

$$\binom{a}{n} = \frac{a(a-1)(a-2)\cdots(a-n+1)}{n!}$$