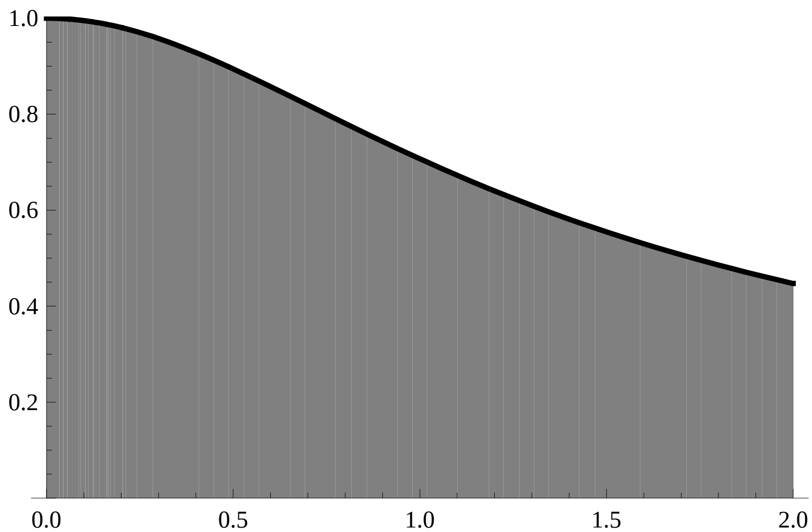
2 June 2008

Quiz 2: Section 6.4, Problem 20

Using the shell method, calculate the volume of the solid obtained by rotating the function $f(x) = \frac{1}{\sqrt{x^2+1}}$ over the interval $[0, 2]$ about the line $x = 0$.

The area that is to be rotated is as follows:



At some arbitrary x-value, our cross-section is a <u>cylinder</u> with a radius of x and a height of $f(x) = \frac{1}{\sqrt{x^2+1}}$. Hence our volume is

$$\begin{aligned} V &= \int_0^2 2\pi x \frac{1}{\sqrt{x^2+1}}\, dx \\ &= 2\pi \int_0^2 \frac{x}{\sqrt{x^2+1}}\, dx \end{aligned}$$

We make the substitution $u = x^2 + 1$ so $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$ and

$$\begin{aligned} 2\pi \int_0^2 \frac{x}{\sqrt{x^2+1}}\, dx &= 2\pi \int_0^2 \frac{x}{\sqrt{u}} \left(\frac{du}{2x}\right) \\ &= \pi \int_0^2 \frac{1}{\sqrt{u}}\, du \\ &= \pi \left(2\sqrt{u} + C\right)\Big|_0^2 \\ &= \pi \left(2\sqrt{x^2+1} + C\right)\Big|_0^2 \\ &= \pi(2\sqrt{5} - 2) \end{aligned}$$