

## Quiz 7: Section 8.1, Problem 9

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Calculate the arc length over the given interval:

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad [1, 2e]$$

Recall the formula for arc length and surface area:

$$\text{Arc Length} = \int_a^b \sqrt{1 + [f'(x)]^2} dx \qquad \text{Surface Area} = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2} dx$$


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Since  $f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ , then  $f'(x) = \frac{1}{2}x - \frac{1}{2x}$ , so  $[f'(x)]^2 = \frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}$ . Therefore, the arc length is

$$\begin{aligned} \int_1^{2e} \sqrt{1 + [f'(x)]^2} dx &= \int_1^{2e} \sqrt{1 + \left(\frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}\right)} dx \\ &= \int_1^{2e} \sqrt{\frac{1}{4}x^2 + \frac{1}{2} + \frac{1}{4x^2}} dx \\ &= \int_1^{2e} \frac{1}{2}x + \frac{1}{2x} dx \\ &= \left(\frac{1}{4}x^2 + \frac{1}{2}\ln x + C\right)\Big|_1^{2e} \\ &= e^2 + \frac{1}{4} + \frac{\ln 2}{2} \end{aligned}$$