Workshop One: Reinventing Geometry

1 Rules for the Workshop

This workshop will be done in randomly-assigned groups of three or four. Each student will get a copy of this workshop, but each group is to turn in only one workshop. It should be neat, clear, and concise. Show all mathematical work. This workshop is intended to be done in-class, but if you need time over the weekend to finish, come talk to me and get my permission.

2 Introduction

We have learned to express areas and volumes as specific definite integrals, which look like

$$\int_a^b f(x)\,dx$$

In this workshop, you will be re-discovering formulas for geometrical shapes whose areas/volumes have been known for thousands of years.

Outline: Each page will present a different two- or three-dimensional figure and its known area or volume. In the case of finding an area, you will need to exhibit a function $f(x)$ and endpoints a and b so that the definite integral

$$\int_a^b f(x)\,dx$$

corresponds to the area of the shape we are dealing with. Calculate the integral to verify that the area formula is indeed correct.

In the case of finding a volume, you will need to show how this volume can be obtained either by discs, washers, or cylindrical shells.

A graph of the function or a representation of the cross-section must be included with each figure.

You may need to consult a table of integrals for this workshop. There is a table of integrals in the back of Rogawski.

3 Rectangles

Show that the area of a rectangle with base b and height h has area bh.

4 Right Triangles

Show that the area of a right triangle with base b and height h has area $\frac{1}{2}bh$.

5 Circles

Show that the area of a circle with radius r has area πr^2.

6 Spheres

Show that the volume of a sphere with radius r has volume $\frac{4}{3}\pi r^3$.

7 Cones

Show that the volume of a cone with a circular base of radius r and height h has volume $\frac{1}{3}\pi r^2 h$.

8 Pyramids

Show that a pyramid of height h whose base is an equilateral triangle of side s has volume $\frac{\sqrt{3}}{12}hs^2$.