Workshop Two: Mad Integration

1 Rules for the Workshop

This workshop will be done in randomly-assigned groups of three or four. Each student will get a copy of this workshop, but each group is to turn in only one workshop. It should be neat, clear, and concise. Show all mathematical work. This workshop is intended to be done in-class, but if you need time over the weekend to finish, come talk to me and get my permission.

2 Introduction

We spent the last week integrating, and we will continue to do so for this workshop. We will mainly show the reduction formulas that were shown in section 7.3 on trigonometric integrals. You'll also do a couple of integrals that aren't too easy to solve.

3 How to integrate $\sec x$

Sometimes we need to modify the integrand to do an integral. Show how to integrate $\int \sec x \; dx$ by multiplying the integrand by $\frac{\sec x + \tan x}{\sec x + \tan x}$ and using substitution afterwards.

4 Reduction Formula 1

Derive the formula

$$\int \sin^n x \, dx = -\sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

Use the formula to find $\int \sin^2 x \, dx$ and $\int \sin^4 x \, dx$.

5 Reduction Formula 2

Derive the formula

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

Use the formula to find $\int \cos^2 x \, dx$ and $\int \cos^4 x \, dx$.

6 Reduction Formula 3

Using integration by parts, derive the formula

$$\int \frac{\sin^n x}{\cos^m x}\, dx = \frac{1}{m-1}\frac{\sin^{n-1} x}{\cos^{m-1} x} - \frac{n-1}{m-1}\int \frac{\sin^{n-2} x}{\cos^{m-2} x}\, dx$$

Use the formula to find $\int \tan^2 x\, dx$ and $\int \tan^4 x\, dx$.

Hint: Finding the right dv is tricky - I'll tell you that you don't want $dv = \sin x\ dx$. However, you have to be able to take the antiderivative of your dv easily!

7 An Alternative Way of Dealing with $\int \sin^m x \cos^n x \; dx$ for m, n Even

(a) If m and n are even, explain how we can reduce the problem of finding $\int \sin^m x \cos^n x \; dx$ into finding integrals of powers of $\cos x$.

(b) It turns out that we can express any power of $\cos x$ as a sum of terms of the form $\cos(nx)$ for some values of n using the trig identity

$$\cos A \cos B = \frac{\cos(A-B) + \cos(A+B)}{2}$$

For example,

$$\cos^2(x) = \cos(x)\cos(x) = \frac{\cos(x-x) + \cos(x+x)}{2} = \frac{1 + \cos(2x)}{2}$$

Following the example above, find similar formulas for $\cos^3(x)$, $\cos^4(x)$ and $\cos^5(x)$. Then find $\displaystyle\int \cos^4(x)\, dx$.