

Workshop Three: ~~Predator-Prey~~ Investigations into ODEs

1 Rules for the Workshop

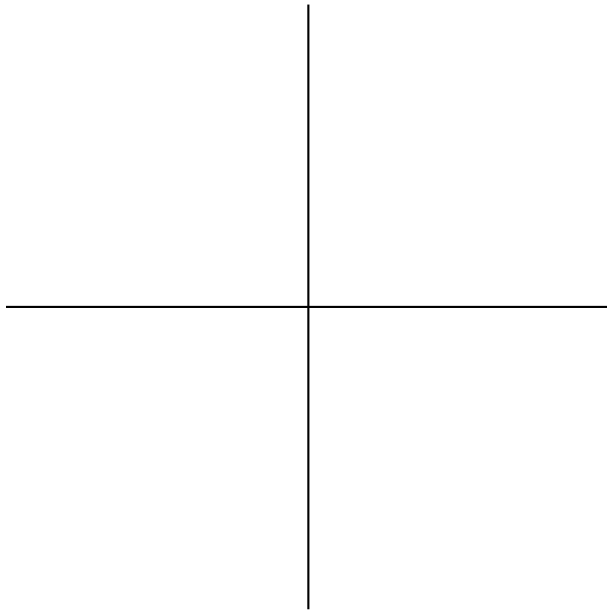
This workshop will be done in randomly-assigned groups of three or four. Each student will get a copy of this workshop, but each group is to turn in only one workshop. It should be neat, clear, and concise. Show all mathematical work. This workshop is intended to be done in-class, but if you need time over the weekend to finish, come talk to me and get my permission.

2 Introduction

We only spent a brief amount of time with differential equations, specifically with the so-called *ordinary differential equations* for reasons which will be explained further in your mathematical career. This workshop is meant to hone your skills in using differential equations, and to play around with the cool-looking *slope fields* (or *direction fields* or whatever).

3 Practice 1

Find a solution of $y' = \frac{y}{x^2 - 1}$ which passes through $(2, 1)$. Simplify and write the solution explicitly as $y = f(x)$. Graph the solution curve, making sure to label the axes. What is the domain of the function describing the solution curve?



4 Practice 2

Consider the differential equations

$$(a) \frac{dy}{dx} = 2x + 3y \quad (b) \frac{dy}{dx} = e^{2x+3y} \quad (c) \frac{dy}{dx} = x^3y^2 \quad (d) \frac{dy}{dx} = x^2 + y^3$$

Two of these are separable. For each of these two separable equations, solve the initial value problem with the initial condition $y(0) = 1$. In each case your solution should be written as $y = f(x)$ where $f(x)$ is a formula. Choose one of the *non-separable* equations and explain carefully why it is *not* separable.

5 Direction Fields 1

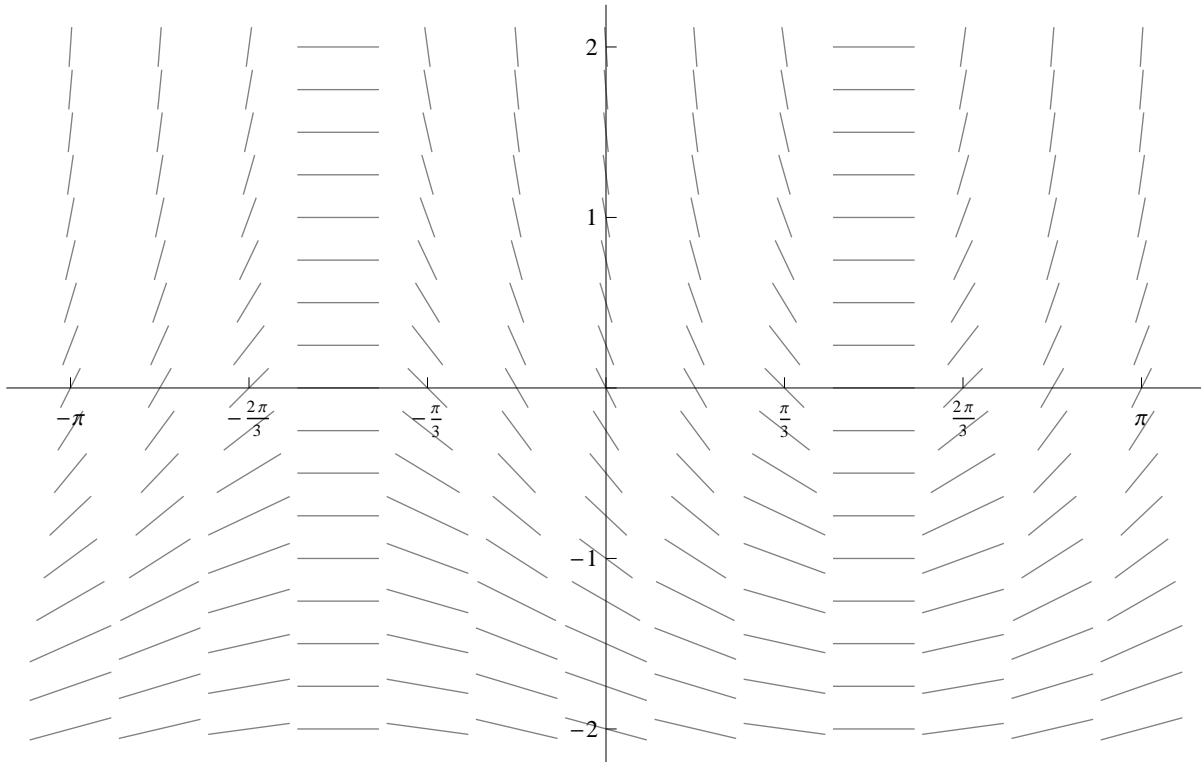
Find the general solution of the differential equation

$$e^{-y} \frac{dy}{dt} + 2 \cos t = 0.$$

a) Does every value of the constant of integration correspond to a solution curve? If not, which C 's do occur?

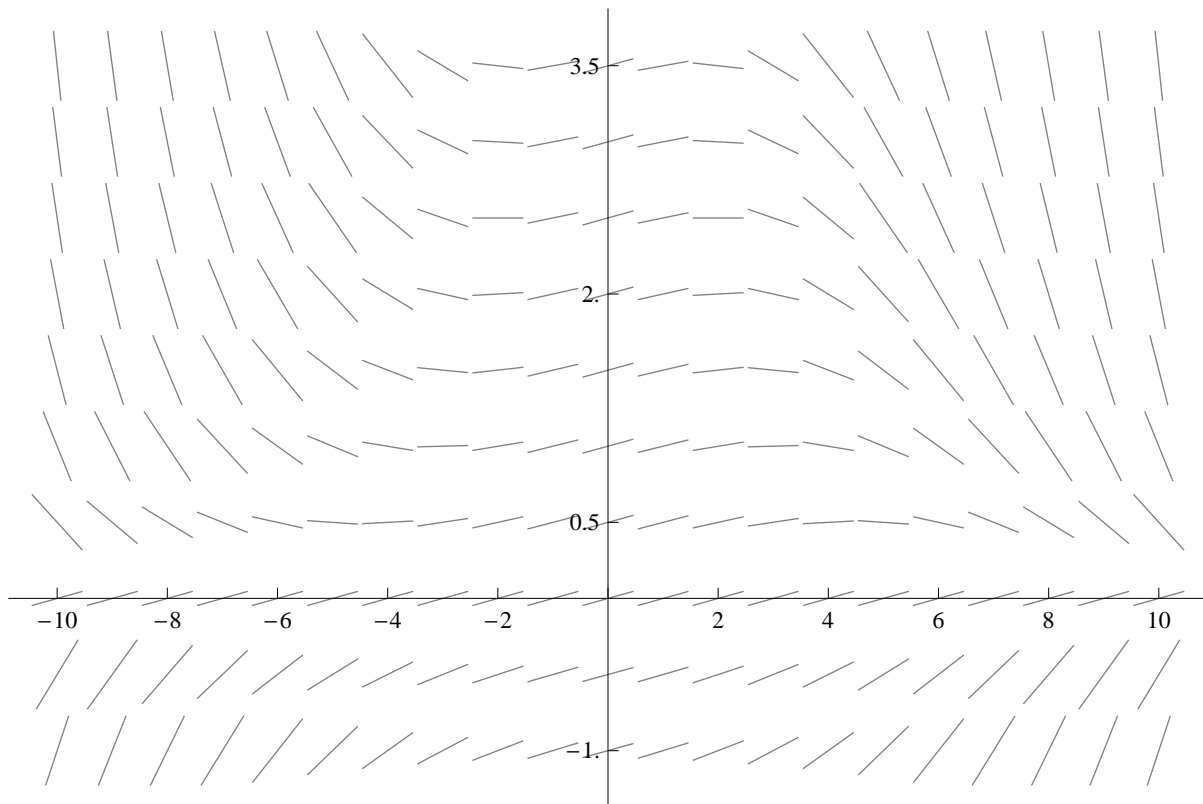
b) Do all the solutions have the same domain? Explain.

c) Below is the direction field associated with this equation. Sketch the solutions corresponding to five values of the constant of integration, C .



6 Direction Fields 2

The following is a direction field for the differential equation $y' = \frac{1}{10} (1 - \frac{1}{10}yx^2)$. Note the axis labels.



- a) Sketch the solution curve which passes through $(0, 1)$ **on the graph above**.
- b) How many critical points does this solution curve seem to have? What types of critical points do they seem to be? If (x_0, y_0) is a critical point, find an exact algebraic relationship between x_0 and y_0 .

Comment The equation *can't* be solved in terms of standard functions. Information from the graph and the differential equation should be used.