Equation Sheet

Trigonometric Identities

$$\begin{aligned}
\sin(s+t) &= \sin s\cos t+\cos s\sin t\\
\sin(s-t) &= \sin s\cos t-\cos s\sin t\\
\cos(s+t) &= \cos s\cos t-\sin s\sin t\\
\cos(s-t) &= \cos s\cos t+\sin s\sin t\\
\tan(s+t) &= \frac{\tan s+\tan t}{1-\tan s\tan t}\\
\tan(s-t) &= \frac{\tan s-\tan t}{1+\tan s\tan t}\\
\sin 2x &= 2\sin x\cos x\\
\cos 2x &= \cos^2 x-\sin^2 x\\
\tan 2x &= \frac{2\tan x}{1-\tan^2 x}\\
\sin\frac{u}{2} &= \pm\sqrt{\frac{1-\cos u}{2}}\\
\cos\frac{u}{2} &= \pm\sqrt{\frac{1+\cos u}{2}}\\
\tan\frac{u}{2} &= \frac{1-\cos u}{\sin u}\\
\sin u\cos v &= \frac{1}{2}[\sin(u+v)+\sin(u-v)]\\
\cos u\sin v &= \frac{1}{2}[\sin(u+v)-\sin(u-v)]\\
\cos u\cos v &= \frac{1}{2}[\cos(u+v)+\cos(u-v)]\\
\sin u\sin v &= \frac{1}{2}[\cos(u-v)-\cos(u+v)]\\
\sin x+\sin y &= 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}\\
\sin x-\sin y &= 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\\
\cos x+\cos y &= 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\\
\cos x-\cos y &= 2\sin\frac{x+y}{2}\sin\frac{x-y}{2}
\end{aligned}$$

Useful Integrals

$$\int \sec^2 u\, du = \tan u + C$$
$$\int \csc^2 u\, du = -\cot u + C$$
$$\int \sec u \tan u\, du = \sec u + C$$
$$\int \csc u \cot u\, du = -\csc u + C$$
$$\int \sec u\, du = \ln|\sec u + \tan u| + C$$
$$\int \csc u\, du = \ln|\csc u - \cot u| + C$$

Forms involving $\sqrt{a^2+u^2}, a > 0$

$$\int \sqrt{a^2+u^2}\, du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln(u+\sqrt{a^2+u^2}) + C$$
$$\int u^2\sqrt{a^2+u^2}\, du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^4}{8}\ln(u+\sqrt{a^2+u^2}) + C$$
$$\int \frac{\sqrt{a^2+u^2}}{u}\, du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$$
$$\int \frac{\sqrt{a^2+u^2}}{u^2}\, du = -\frac{\sqrt{a^2+u^2}}{u} + \ln(u+\sqrt{a^2+u^2}) + C$$
$$\int \frac{du}{\sqrt{a^2+u^2}} = \ln(u+\sqrt{a^2+u^2}) + C$$
$$\int \frac{u^2\, du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln(u+\sqrt{a^2+u^2}) + C$$
$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right| + C$$
$$\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2 u} + C$$
$$\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$$

Forms involving $\sqrt{a^2-u^2}, a > 0$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

$$\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{1}{a^2u}\sqrt{a^2-u^2} + C$$

$$\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$$

Forms involving $\sqrt{u^2-a^2}, a > 0$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$$

$$\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{|u|} + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln|u+\sqrt{u^2-a^2}| + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln|u+\sqrt{u^2-a^2}| + C$$

$$\int \frac{u^2\,du}{\sqrt{u^2-a^2}} = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$$

$$\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$$

$$\int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$$

Integral Approximations

If we split up our interval $[a,b]$ into n subintervals

$$[x_0,x_1],[x_1,x_2],\ldots,[x_{n-1},x_n] \qquad x_0 < x_1 < \cdots < x_n$$

Midpoint Rule	$\int_a^b f(x)\,dx \approx \Delta x[f(\overline{x_1}) + f(\overline{x_2}) + \cdots + f(\overline{x_n})]$
Trapezoidal Rule	$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$
Simpson's Rule (n even)	$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)]$