

Solutions - 10.4

5. The area is

$$\int_0^\pi \frac{1}{2} r^2 d\theta = \frac{1}{2} \int_0^\pi \theta^2 d\theta = \frac{1}{2} \left(\frac{\theta^3}{3} \right) \Big|_0^\pi = \frac{\pi^3}{6}$$

10. Sketching it is a graphy, but the area will be

$$\int_0^{2\pi} 3(1 + \cos \theta) d\theta = 3(\theta + \sin \theta) \Big|_0^{2\pi} = 6\pi$$

14. Sketching it is a graphy, but the area will be (why is the upper bound not 2π ?)

$$\int_0^\pi 2 + \cos 2\theta d\theta = (2\theta + \frac{1}{2} \sin 2\theta) \Big|_0^\pi = 2\pi$$

21. The inner loop happens from $\theta = \frac{7\pi}{6}$ to $\theta = \frac{11\pi}{6}$. So our area is

$$\begin{aligned} \frac{1}{2} \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (1 + 2 \sin \theta)^2 d\theta &= \frac{1}{2} \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} 1 + 4 \sin \theta + 4 \sin^2 \theta d\theta \\ &= \frac{1}{2} (\theta - 4 \cos \theta + 2\theta - \sin 2\theta) \Big|_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \\ &= \pi - \frac{3\sqrt{3}}{2} \end{aligned}$$

23. So we need to find

$$\begin{aligned} \frac{1}{2} \int_0^\pi (4 \sin \theta)^2 d\theta - \frac{1}{2} \int_0^\pi 2^2 d\theta &= (4\theta - 2 \sin 2\theta) \Big|_0^\pi - 2\theta \Big|_0^\pi \\ &= 4\pi - 2\pi \\ &= 2\pi \end{aligned}$$

28. Note that the curves meet up at $\theta = \pm \frac{\pi}{3}$, so the integral we need to compute is

$$\begin{aligned} \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (1 - 2 \cos \theta)^2 d\theta &= \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1 - 4 \cos \theta + 4 \cos^2 \theta d\theta \\ &= \frac{1}{2} (\theta - 4 \sin \theta + 2\theta + \sin 2\theta) \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \\ &= \pi - \sqrt{32} \end{aligned}$$

45. We have

$$\begin{aligned}\int_0^{\frac{\pi}{3}} \sqrt{9 \sin^2 \theta + 9 \cos^2 \theta} \, d\theta &= \int_0^{\frac{\pi}{3}} 3 \, d\theta \\ &= \pi\end{aligned}$$