How to do 7.3, problem 26

I won't go through all the details until the end, but I'll get you started really well. We have to find

$$\int \frac{x^2}{\sqrt{4x - x^2}}\, dx$$

Now, we *want* the square root to look like $a^2 - x^2$ or something like that, so we'll need to complete the square. Note that

$$4x - x^2 = -(x-2)^2 + 4$$

So we have

$$\int \frac{x^2}{\sqrt{4 - (x-2)^2}}\, dx$$

Now, if we *wanted* to do the substitution $x - 2 = \sin\theta$, we would have to take care of the x^2 in the numerator. But we can't simply get x^2 - we have to do $(x-2)^2$, which is equal to $x^2 - 4x + 4$. So how to deal with that? Well, we "make" the numerator $x^2 - 4x + 4$ as follows:

$$\begin{aligned}\int \frac{x^2}{\sqrt{4-(x-2)^2}}\, dx &= \int \frac{x^2 - 4x + 4(+4x - 4)}{\sqrt{4-(x-2)^2}}\, dx \\ &= \int \frac{x^2 - 4x + 4}{\sqrt{4-(x-2)^2}}\, dx + \int \frac{4x}{\sqrt{4-(x-2)^2}}\, dx - \int \frac{4}{\sqrt{4-(x-2)^2}}\, dx\end{aligned}$$

Now, we have three integrals to do. For the first integral, we can do what we initially wanted. For the second integral, we can do a simple substitution. For the third integral, we can use the same substitution as the first, and we won't have a problem from there!

I highly suggest you go on from there and solve the integral.