

Solutions: 6.5

2. The average value is

$$\begin{aligned}\frac{1}{4-1} \int_1^4 \frac{1}{x} dx &= \frac{1}{3} \ln x \Big|_1^4 \\ &= \frac{1}{3} \ln 4\end{aligned}$$

5. The average value is

$$\begin{aligned}\frac{1}{5-0} \int_0^5 te^{-t^2} dt &= \frac{1}{5} \left(\frac{-e^{-t^2}}{2} \right) \Big|_0^5 \\ &= \frac{1}{5} \left(\frac{-e^{-25}}{2} + \frac{e^0}{2} \right) \\ &= \frac{1 - e^{-25}}{10}\end{aligned}$$

9.(a) The average value is

$$\begin{aligned}\frac{1}{5-2} \int_2^5 (x-3)^2 dx &= \frac{1}{3} \left(\frac{(x-3)^3}{3} \right) \Big|_2^5 \\ &= \frac{1}{3} \left(\frac{4}{3} - \frac{1}{3} \right) \\ &= \frac{1}{3}\end{aligned}$$

(b) We need to find x such that

$$(x-3)^2 = \frac{1}{3}$$

But that is easy:

$$\begin{aligned}(x-3)^2 &= \frac{1}{3} \\ x-3 &= \frac{\sqrt{3}}{3} \\ x &= 3 + \frac{\sqrt{3}}{3}\end{aligned}$$

(c) Do it yourself!

19. We need to compute the integral

$$\begin{aligned}\frac{1}{8} \int_0^8 \frac{12}{\sqrt{x+1}} dx &= \frac{1}{8} (24\sqrt{x+1}) \Big|_0^8 \\ &= \frac{1}{8} (24\sqrt{9} - 24\sqrt{1}) \\ &= 6\end{aligned}$$