Solutions: 7.2

2. We have

$$\begin{aligned}
\int \sin^6 x \cos^3 x \, dx &= \int \sin^6 x (1-\sin^2 x)\cos x \, dx \\
&= \int \sin^6 x \cos x \, dx - \int \sin^8 x \cos x \, dx \\
&= \frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C
\end{aligned}$$

7. Recall that $\cos 2\theta = 2\cos^2\theta - 1$, so we have

$$\begin{aligned}
\int \cos^2\theta \, d\theta &= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta \\
&= \frac{1}{2}(\theta + \frac{1}{2}\sin 2\theta)
\end{aligned}$$

So

$$\int_0^{\pi/2} \cos^2\theta \, d\theta = \frac{1}{2}(\theta + \frac{1}{2}\sin 2\theta)|_0^{\pi/2} = \frac{\pi}{4}$$

19. We have

$$\begin{aligned}
\int \frac{1-\sin x}{\cos x}\, dx &= \int \sec x \, dx - \int \tan x \, dx \\
&= \ln|\sec x + \tan x| - \ln|\sec x| + C
\end{aligned}$$

23. We will use integration by parts, with

$$\begin{array}{ll}
u = \sin x & v = \sec x \\
du = \cos x \, dx & dv = \frac{\sin x}{\cos^2 x}\, dx
\end{array}$$

And this gives us

$$\begin{aligned}
\int \tan^2 x \, dx &= \frac{\sin x}{\cos x} - \int 1 \, dx \\
&= \tan x - x + C
\end{aligned}$$

24. Recall that $\tan^2 x = 1 - \sec^2 x$ so

$$\begin{aligned}
\int \tan^4 x \, dx &= \int \tan^2 x(1-\sec^2 x)\, dx \\
&= \int \tan^2 x \, dx - \int \tan^2 x \sec^2 x \, dx \\
&= \tan x - x - \frac{\tan^3 x}{3}
\end{aligned}$$

59. Using the slicing method, we obtain as cross-sections circles of radius $\sin x$ and area $\pi \sin^2 x$. Therefore the integral we need is

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \pi \sin^2 x \, dx &= \pi \int_0^{\frac{\pi}{2}} \sin^2 x \, dx \\
&= \frac{\pi}{2}(x - \frac{1}{2}\sin 2x)|_0^{\frac{\pi}{2}} \\
&= \frac{\pi}{4}
\end{aligned}$$

64. We need to find

$$\sqrt{\frac{1}{\frac{1}{60}}\int_0^{1/60}(155\sin(120\pi t))^2\,dt} = 310\sqrt{15}\sqrt{\int_0^{1/60}\sin^2(120\pi t)\,dt}$$

Recall that

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

So

$$\sin^2(120\pi t) = \frac{1-\cos(240\pi t)}{2}$$

and so

$$\int \sin^2(120\pi t)\,dt = \int \frac{1}{2}\,dt - \frac{1}{2}\int \cos(240\pi t)\,dt = \frac{x}{2} - \frac{1}{480\pi}\sin(240\pi t)$$

And note that

$$\left(\frac{x}{2} - \frac{1}{480\pi}\sin(240\pi t)\right)\Bigg|_0^{1/60} = \frac{1}{120}$$

and so we have

$$310\sqrt{15}\sqrt{\int_0^{1/60}\sin^2(120\pi t)\,dt} = 310\sqrt{15}\sqrt{\frac{1}{120}} \approx 109.6015511$$

which is our final answer.
(b) From the work done in part (a), we see that the equation we need to solve is

$$2A\sqrt{15}\sqrt{\frac{1}{120}} = 220$$

which gives us $A \approx 311.1269837$.