Solutions: 7.5

1.

$$\int \frac{\sin x + \sec x}{\tan x}\,dx = \int \cos x + \csc x\,dx = \sin x + \ln|\csc x - \cot x| + C$$

4. Let $u = \frac{x^2}{\sqrt{3}}$. Then $du = \frac{2x}{\sqrt{3}}\,dx$ so by substitution we get

$$\int \frac{x}{\sqrt{3 - x^4}}\,dx = \frac{\sqrt{3}}{2}\int \frac{du}{\sqrt{3 - 3u^2}} = \frac{1}{2}\int \frac{du}{\sqrt{1 - u^2}} = \frac{1}{2}\sin^{-1} u = \frac{1}{2}\sin^{-1}\frac{x^2}{3}$$

8. We set up partial fractions:

$$\frac{x-1}{x^2 - 4x - 5} = \frac{A}{x-5} + \frac{B}{x+1}$$

which yields the solution $A = 2/3$ and $B = 1/3$. Therefore,

$$\int \frac{x-1}{x^2 - 4x - 5}\,dx = \frac{2}{3}\int \frac{1}{x-5}\,dx + \frac{1}{3}\int \frac{1}{x+1}\,dx = \frac{2}{3}\ln|x-5| + \frac{2}{3}\ln|x+1| + C$$

9. The denominator is irreducible so we cannot use partial fractions. Note, however, that

$$\int \frac{x-1}{x^2 - 4x + 5}\,dx = \int \frac{x-2}{x^2 - 4x + 5}\,dx + \int \frac{1}{x^2 - 4x + 5}\,dx$$

For the first integral, we can make the substitution $u = x^2 - 4x + 5$ to get $du = 2x - 4\,dx = 2(x-2)\,dx$ and so

$$\int \frac{x-2}{x^2 - 4x + 5}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|x^2 - 4x + 5|$$

For the second integral, note that $x^2 - 4x + 5 = (x-2)^2 + 1$ so

$$\int \frac{1}{x^2 - 4x + 5}\,dx = \int \frac{1}{(x-2)^2 + 1}\,dx = \tan^{-1}(x-2)$$

so combining it all we obtain

$$\int \frac{x-1}{x^2 - 4x + 5} = \frac{1}{2}\ln|x^2 - 4x + 5| + \tan^{-1}(x-2) + C$$

19. If we let $u = e^{e^x}$ then $du = e^{x + e^x}\,dx$ so

$$\int e^{x + e^x}\,dx = \int du = u = e^{e^x} + C$$

22. If we let

$$\begin{array}{ll} u = \sin^{-1} x & v = \frac{x^2}{2} \\ du = \frac{1}{\sqrt{1-x^2}}\,dx & dv = x\,dx \end{array}$$

then

$$\int x \sin^{-1} x\,dx = \frac{x^2}{2}\sin^{-1} x - \frac{1}{2}\int \frac{x^2\,dx}{\sqrt{1-x^2}} = \frac{x^2}{2}\sin^{-1} x - \frac{1}{2}\left[-\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right] + C$$

29.

$$\int \frac{3w - 1}{w + 2}\,dw = \int \frac{3(w+2)}{w+2}\,dw - \int \frac{7}{w+2}\,dw = 3w - 7\ln|w+2| + C$$

So

$$\int_0^5 \frac{3w-1}{w+2}\,dw = [3w - 7\ln|w+2|]|_0^5 = 15 - 7(\ln 7 - \ln 2)$$

57. If we let

$$\begin{array}{ll} u = x & v = \frac{3}{4}(x+c)^{\frac{4}{3}} \\ du = dx & dv = \sqrt[3]{x+c} \end{array}$$

then

$$\int x\sqrt[3]{x+c}\,dx = \frac{3}{4}x(x+c)^{\frac{4}{3}} - \int \frac{3}{4}(x+c)^{\frac{4}{3}}\,dx = \frac{3}{4}x(x+c)^{\frac{4}{3}} - \frac{9}{28}(x+c)^{\frac{7}{3}} + C$$

74. Note that

$$\frac{1}{e^x - e^{-x}} = \frac{e^x}{e^{2x}-1}$$

so if we make the substitution $u = e^x$ then $du = e^x\,dx$ and

$$\int \frac{1}{e^x - e^{-x}}\,dx = \int \frac{e^x}{e^{2x}-1}\,dx = \int \frac{1}{u^2+1}\,du = \tan^{-1} u = \tan^{-1} e^x + C$$