

Quiz 10 - Math 152

Suppose we are dealt with the differential equation

$$y' = -\frac{x}{y}$$

If we rewrite it as

$$\frac{dy}{dx} = -\frac{x}{y}$$

then we can change things around to

$$y \, dy = -x \, dx$$

(a) Put an integral sign in front of both sides of the equation, and solve with the initial condition $y(0) = 4$.

(b) Show that the answer in (a) is indeed a solution to the differential equation $y' = -\frac{x}{y}$.

Solutions.

(a) If we integrate, we get

$$\begin{aligned} \int y \, dy &= \int -x \, dx \\ \frac{y^2}{2} &= -\frac{x^2}{2} + C \end{aligned}$$

Using the fact that $y(0) = 4$, we get $C = 8$. Hence, we continue:

$$\begin{aligned} \frac{y^2}{2} &= -\frac{x^2}{2} + 8 \\ y &= \sqrt{16 - x^2} \end{aligned}$$

(b) Note that if $y = \sqrt{16 - x^2}$ then

$$\frac{dy}{dx} = \frac{1}{2\sqrt{16 - x^2}}(-2x) = -\frac{x}{\sqrt{16 - x^2}} = -\frac{x}{y}$$

so we're done!