

Quiz 14 - Math 152

For each of the following four series, say whether it is convergent or divergent.

1.

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 1}$$

This is convergent, because

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 1} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$$

2.

$$\sum_{n=1}^{\infty} \frac{5}{2 + 3^n}$$

This is convergent, because

$$\sum_{n=1}^{\infty} \frac{5}{2 + 3^n} \leq 5 \sum_{n=1}^{\infty} \frac{1}{3^n} < \infty$$

the last series being convergent because it is a geometric series with ratio $\frac{1}{3}$.

3.

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2 + 1}$$

This is convergent because

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2 + 1} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$$

4.

$$\sum_{n=1}^{\infty} \frac{1}{2n + 3}$$

This is divergent, because

$$\sum_{n=1}^{\infty} \frac{1}{2n + 3} \geq \frac{1}{5n} = \infty$$