

5 July 2006

Quiz 17 - Math 152

Recall that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

With that in mind, give the power series for the following:

(a)

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \dots = 1 - x^2 + x^4 - x^6 + x^8 - \dots$$

(b)

$$\frac{1}{(1-x)^2} = \frac{d}{dx} \frac{1}{1-x} = \frac{d}{dx} (1 + x + x^2 + x^3 + \dots) = 1 + 2x + 3x^2 + 4x^3 + \dots$$

(c)

$$\frac{x}{1-x} = x(1 + x + x^2 + x^3 + \dots) = x + x^2 + x^3 + x^4 + \dots$$

(d)

$$\ln\left(\frac{1}{1-x}\right) = -\int \frac{1}{1-x} dx = -\int (1 + x + x^2 + x^3 + \dots) dx = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots$$

(e)

$$\tan^{-1} x = \int \frac{1}{1+x^2} dx = \int (1 - x^2 + x^4 - x^6 + x^8 - \dots) dx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$