Quiz 18 - Math 152

Find the intervals of convergence for the following:

$$\begin{aligned} \tan^{-1} x &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \end{aligned}$$

Solutions.

For $\tan^{-1} x$, using the ratio test we have

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|x^2 \frac{2n+1}{2n+3}\right|$$

So

$$\lim_{n\to\infty} \left|x^2 \frac{2n+1}{2n+3}\right| = |x^2|$$

So the radius of convergence equals 1. At the endpoints, $x = 1$ and $x = -1$, we converge both times as the series is alternating and the terms are going to 0. Therefore the interval of convergence is $[-1, 1]$.

For $\sin x$, using the ratio test we have

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|x^2 \frac{1}{(2n+3)(2n+2)}\right|$$

So

$$\lim_{n\to\infty} \left|x^2 \frac{1}{(2n+3)(2n+2)}\right| = 0$$

and so $\sin x$ converges everywhere.