

18 July 2006

Quiz 22 - Math 152

1. Find the points on the following curve where the slope of the tangent line is either horizontal or vertical:

$$\begin{aligned}x &= 2t^3 + 3t^2 - 12t \\y &= 2t^3 + 3t^2 + 1\end{aligned}$$

2. Set up, but do not evaluate, the integral for the following arc length:

$$\begin{aligned}x &= 1 + e^t \\y &= t^2 \\-3 &\leq t \leq 3\end{aligned}$$

Solutions.

1. We have

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t^2 + 6t - 12}{6t^2 + 6t} = \frac{6(t+2)(t-1)}{6t(t+1)}$$

So we see that we are horizontal when $t = -2$ and $t = 1$, which corresponds to the points $(20, -3)$ and $(-7, 6)$. Likewise, we are vertical when $t = 0$ and $t = -1$, which corresponds to the points $(0, 1)$ and $(13, 2)$.

2. We have

$$A = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_{-3}^3 \sqrt{(e^t)^2 + (2t)^2} dt$$