

19 July 2006

### Quiz 23 - Math 152

1. Find the distance between the two points  $(1, \frac{\pi}{6})$  and  $(3, \frac{3\pi}{4})$ , where the two points are given in polar coordinates.
2. The graph of  $r = \cos 2\theta$  is in the shape of a four-leaf clover. What is the area of one of the leaves?

#### Solutions.

1. We can convert to cartesian coordinates:

$$\begin{aligned} (1, \frac{\pi}{6}) &\rightarrow (\frac{\sqrt{3}}{2}, \frac{1}{2}) \\ (3, \frac{3\pi}{4}) &\rightarrow (-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}) \end{aligned}$$

So now, with the distance formula, the distance between the two is

$$\sqrt{\left(\frac{\sqrt{3}}{2} + \frac{3\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2} - \frac{3\sqrt{2}}{2}\right)^2}$$

2. The leaf is bounded by  $\theta = -\frac{\pi}{4}$  and  $\theta = \frac{\pi}{4}$ . Therefore, the integral we are to solve is

$$\frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 2\theta \, d\theta = \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 + \cos 4\theta}{2} \, d\theta = \frac{\pi}{8}$$