05 June 2006

Quiz 3 - Math 152

1. What is the average value of $f(x) = x^2 - 6x + 5$ on the interval $[4, 7]$? What value of x in this interval achieves this average value?

2. Recall: a function f is odd if $f(-x) = -f(x)$ for all x. Show that if f is odd and continuous on $[-2, 2]$, then the average value of f is 0.

Solutions.

1. For the average value, we need

$$\begin{aligned}
\frac{1}{3}\int_4^7 x^2 - 6x + 5\,dx &= \frac{1}{3}\left(\frac{x^3}{3} - 3x^2 + 5x\right)\Bigg|_4^7 \\
&= \frac{1}{3}\left(\left(\frac{343}{3}\right) - 147 + 35\right) - \left(\frac{64}{3} - 48 + 20\right) \\
&= 3
\end{aligned}$$

So now, to find the desired x-value, we need to solve

$$\begin{aligned}
x^2 - 6x + 5 &= 3 \\
x^2 - 6x + 2 &= 0 \\
x &= \frac{6 \pm \sqrt{36 - 8}}{2} \\
x &= 3 \pm \sqrt{7}
\end{aligned}$$

Note, though, that only $3 + \sqrt{7}$ falls in the interval $[4, 7]$, so our final answer is $x = 3 + \sqrt{7}$.

2. We have the following:

$$\begin{aligned}
\frac{1}{4}\int_{-2}^{2} f(x)\,dx &= \frac{1}{4}\int_{-2}^{0} f(x)\,dx + \frac{1}{4}\int_{0}^{2} f(x)\,dx \\
&= \frac{1}{4}\int_{-2}^{0} -f(-x)\,dx + \frac{1}{4}\int_{0}^{2} f(x)\,dx \\
&= -\frac{1}{4}\int_{0}^{2} f(x)\,dx + \frac{1}{4}\int_{0}^{2} f(x)\,dx \\
&= 0
\end{aligned}$$