

Quiz 5

Recall that at the end of class yesterday we attempted to find

$$\int \tan^4 x \sec x \, dx$$

and we became left with trying to calculate

$$\int \sec^3 x \, dx \quad \text{and} \quad \int \sec^5 x \, dx$$

1. Use integration by parts with $dv = \sec^2 x \, dx$ to find $\int \sec^3 x \, dx$. (HINT: $\tan^2 x = \sec^2 x - 1$).
2. Use part (1) to find $\int \sec^5 x \, dx$.

Solutions.

1. With the hint, we use

$$\begin{aligned} u &= \sec x & v &= \tan x \\ du &= \sec x \tan x \, dx & dv &= \sec^2 x \, dx \end{aligned}$$

and we get

$$\begin{aligned} \int \sec^3 x \, dx &= \sec x \tan x - \int \sec x \tan^2 x \, dx \\ &= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx \\ &= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx \\ &= \sec x \tan x - \int \sec^3 x \, dx + \ln |\sec x + \tan x| \end{aligned}$$

so solving for $\int \sec^3 x \, dx$ we get

$$\int \sec^3 x \, dx = \frac{1}{2}(\sec x \tan x + \ln |\sec x + \tan x|)$$

2. Again, we use parts with

$$\begin{aligned} u &= \sec^3 x & v &= \tan x \\ du &= 3 \sec^2 x \sec x \tan x \, dx & dv &= \sec^2 x \, dx \end{aligned}$$

and we get

$$\begin{aligned} \int \sec^5 x \, dx &= \tan x \sec^3 x - 3 \int \sec^3 x \tan^2 x \, dx \\ &= \tan x \sec^3 x - 3 \int \sec^3 x (\sec^2 x - 1) \, dx \\ &= \tan x \sec^3 x - 3 \int \sec^5 x \, dx + 3 \int \sec^3 x \, dx \\ &= \tan x \sec^3 x - 3 \int \sec^5 x \, dx + \frac{3}{2}(\sec x \tan x + \ln |\sec x + \tan x|) \end{aligned}$$

so solving for $\int \sec^5 x \, dx$ we get

$$\int \sec^5 x \, dx = \frac{1}{4} \tan x \sec^3 x + \frac{3}{8}(\sec x \tan x + \ln |\sec x + \tan x|)$$