Quiz 6

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1. Find

$$\int \frac{t}{\sqrt{t^2-16}}\,dt$$

2. Find

$$\int \frac{1}{\sqrt{t^2-16}}\,dt$$

3. (extra credit - 2pt) Without doing *any* calculations, tell what

$$\int_{-4}^{4} \sqrt{16-x^2}\,dx$$

is, and explain why.

Solutions.

1. We can do a basic substitution. If we let $u = t^2 - 16$, then $du = 2t\,dt$ and we get

$$\int \frac{t}{\sqrt{t^2-16}}\,dt = \int \frac{1}{2\sqrt{u}}\,du = \sqrt{u} = \sqrt{t^2-16}$$

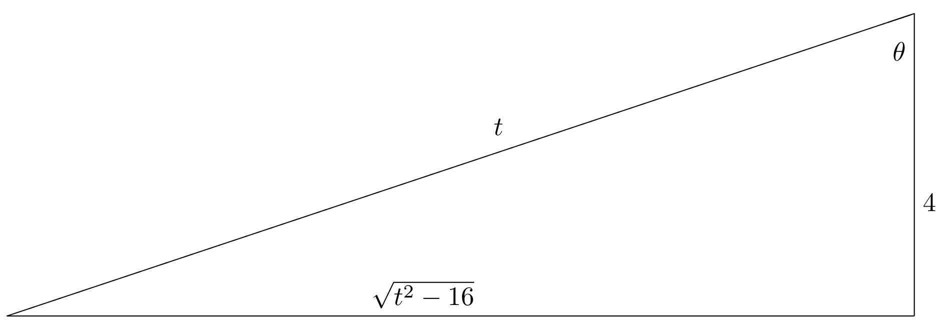
2. We have to do trig substitution for this one. We let

$$\begin{aligned} t &= 4\sec\theta \\ dt &= 4\sec\theta\tan\theta \end{aligned}$$

and we get

$$\begin{aligned} \int \frac{1}{\sqrt{t^2-16}}\,dt &= \int \frac{4\sec\theta\tan\theta}{\sqrt{16\sec^2\theta-16}}\,d\theta \\ &= \int \frac{\sec\theta\tan\theta}{\tan\theta}\,d\theta \\ &= \int \sec\theta\,d\theta \\ &= \ln|\sec\theta+\tan\theta| \\ &= \ln|\sec\sec^{-1}\frac{t}{4}+\tan\sec^{-1}\frac{t}{4}| \end{aligned}$$

so our task is now to compute $\tan\sec^{-1}\frac{t}{4}$. The appropriate triangle is as follows:



So from this, we see that $\tan\sec^{-1}\frac{t}{4} = \frac{\sqrt{t^2-16}}{4}$. So we finally have

$$\int \frac{1}{\sqrt{t^2-16}}\,dt = \ln\left|\frac{t}{4} + \frac{\sqrt{t^2+16}}{4}\right|$$

3. The graph of the function $f(x) = \sqrt{16-x^2}$ is a semicircle from -4 to 4. Therefore,

$$\int_{-4}^{4} \sqrt{16-x^2}\,dx$$

is the area of this semicircle, which is 8π.