

Quiz 7 - Math 152

1. Put the following in partial fraction form, but do not solve:

$$\frac{1}{(x-2)^3(x^2+x+1)^3x}$$

2. Find $\int \ln(x^2+1) dx$.

Solutions.

1.

$$\frac{1}{(x-2)^3(x^2+x+1)^3x} = \frac{A}{x-2} + \frac{B}{(x-1)^2} + \frac{C}{(x-2)^3} + \frac{Dx+E}{x^2+x+1} + \frac{Fx+G}{(x^2+x+1)^2} + \frac{Hx+I}{(x^2+x+1)^3} + \frac{J}{x}$$

2. We first do integration by parts. If we let

$$\begin{aligned} u &= \ln(x^2+1) & v &= x \\ du &= \frac{2x}{x^2+1} dx & dv &= 1 dx \end{aligned}$$

then we have

$$\int \ln(x^2+1) dx = x \ln(x^2+1) - 2 \int \frac{x^2}{x^2+1} dx$$

Now we are in a partial fraction situation. Note that

$$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$$

so we get

$$\int \frac{x^2}{x^2+1} dx = \int 1 dx - \int \frac{1}{x^2+1} dx = x - \tan^{-1} x$$

and so, finally, we have

$$\int \ln(x^2+1) dx = x \ln(x^2+1) - 2(x - \tan^{-1} x)$$